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WADD TECHNICAL REPORT 61-133
VOLUME III

PASSIVE AERODYNAMIC ATTITUDE STABILIZATION OF NEAR EARTH SATELLITES

Volume III
MATHEMATICAL TECHNIQUES AND COMPUTER PROGRAM

O. C. JUELICH

NORTH AMERICAN AVIATION, INC.
COLUMBUS, OHIO

JULY 1961

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AERONAUTICAL SYSTEMS DIVISION

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FLIGHT DYNAMICS LABORATORY
CONTRACT Nr. AF 33(616)-7100
PROJECT Nr. 1366
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AERONAUTICAL SYSTEMS DIVISION
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UNITED STATES AIR FORCE
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FOREWORD

The research reported herein was performed by North American Aviation, Inc., Columbus, Ohio for the Hypersonic Flight Section, Flight Branch of the Flight Dynamics Laboratory, Wright Air Development Division. The work was accomplished under Air Force Contract No. AF 33(616)-7100, Project No. 1366, Task No. 13967, "A Study of Aerodynamically Oriented and Stabilized Satellites." This research was carried out by the Engineering Research and Aerothermodynamics Development Groups of the Columbus Division, North American Aviation, Inc., with Dr. D. M. Schrello, Engineering Research Group, as Project Engineer. Mr. Joseph Ondrejka, Flight Dynamics Laboratory, was WADD Project Engineer.

The results of this study are reported in a series of three volumes of which this is Volume III. The other reports in this series are:

VOLUME I: "Librations due to Combined Aerodynamic and Gravitational Torques," by D. M. Schrello

VOLUME II: "Aerodynamic Analysis," by Paul H. Davison

ABSTRACT

A computer program is presented which integrates numerically the pitch equation of an aerodynamically stabilized satellite. The mathematical theory of the linear second order equation with periodic coefficients and the integration procedure are discussed to clarify the structure and application of the program.

PUBLICATION REVIEW

This report has been reviewed and is approved.

FOR THE COMMANDER:

William C. Nielsen

WILLIAM C. NIELSEN
Colonel, USAF
Chief, Flight Dynamics Laboratory

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LIST OF SYMBOLS

A, B, C, D	Coefficients used in the derivation of Eq. (32)
a, b	Initial condition constants
a_n	The nth coefficient of Taylor's Series
F	Number of a card field
f	Function generated in the card-punch program
h	Altitude, increment in t
I	Identity matrix
i	Orbit inclination, Counting index
M	Inertia parameter
n	Counting index, integral multiplier
P, P_p, Q, Q_p	Aerodynamic parameters
r	Distance from center of earth
r_E	Mean radius of earth
S_n	Matrix defined in Eq. (22)
s	Constant appearing in error term of Taylor's series
t	Independent variable
V	Velocity of vehicle in inertial space
V_R	Velocity of vehicle relative to atmosphere
v	True anomaly of vehicle
W	Matrix whose determinant is the Wronskian of the system (y_1, y_2)
$\boldsymbol{w}$	Wronskian of the system (y_1, y_2)
w	Eigenvalue bound for W , defined in Eq. (23)
w_P, w_1, w_2, w_y	Weighting constants in card-punch program
y	Solution of differential equation, Eqs. (5) or (6)
α	Angle of attack

α, β, γ	Periodic functions of t
Γ, Γ_ϕ	Aerodynamic parameters
γ	Inclination of orbit to local horizontal
ϵ	Eccentricity, error
θ	Pitch angle relative to local horizontal
ρ	Atmospheric density
σ_{ni}	Eigenvalues of matrix S_n
ω_E	Earth rotation rate
ω	Eigenvalue of the matrix W

Subscripts (except counting indices)

e	even part
n	operator defined in Eq. (13)
o	odd part
p	perigee
p	periodic, calculated from predictor formula
s	calculated from Simpson's rule
0	particular solution
$1, 2$	homogeneous solution

Notation

$\dot{x}$	Derivative with respect to t
x'	Derivative with respect to ν
$\bar{x}$	Maximum magnitude, see Eq. (11)
x^*	An instance of x
Δx	An increment of x
$\frac{d}{dx}$	Differentiation
$\cong$	Approximate equality
$\equiv$	Identity

INTRODUCTION

The equation of satellite pitching motion derived in Volume I of this series of reports is:

$$\theta'' + 2Q\theta' + \{3M[1-\epsilon\cos v] + P\}\theta = 2Q + \gamma P \tag{1}$$

where v , the true anomaly, is the independent variable,

θ is the pitch angle from the local horizontal,

ϵ is the orbital eccentricity,

M is the inertia ratio parameter,

and the remaining quantities are defined by the following relations:

$$\gamma = \frac{\epsilon \sin v}{1+\epsilon \cos v} , \tag{2}$$

$$P = P_p \,(\rho/\rho_p)(V_R/V)^2 [1 + 2\epsilon(1-\cos v)] ,$$

$$Q = Q_p \,(\rho/\rho_p)(V_R/V)[1-\epsilon(1-\cos v)] - \epsilon \sin v ,$$

in which

$$(V_R/V) = 1 - (\omega_E/\dot{v}_p)\cos i ,$$

$$\dot{v}_p = \frac{1}{r_p}\sqrt{\frac{\mu(1+\epsilon)}{r_p}} , \tag{3}$$

the subscript p denotes perigee conditions,

μ is the product of the universal gravitational constant and the mass of the earth,

ω_E is the Earth's rotation rate,

i is the orbit inclination,

ρ , the atmospheric density, is a tabular function of the altitude h , see for instance Minzner, (Ref. 1). The altitude h in turn is defined by

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$$h = r - r_E \tag{4}$$

with r_E the mean earth radius, and

$$r = r_p(1+\epsilon)/(1+\epsilon \cos \nu).$$

The quantities $\mathcal{P}_p$, $\mathcal{Q}_p$ are defined by

$$\mathcal{P}_p = \Gamma \rho_p \, r_p^2 / 2$$

$$\mathcal{Q}_p = \Gamma_q \, \rho_p \, r_p / 8$$

where Γ is a pitching moment parameter, and

Γ_q is a damping in pitch parameter.

These parameters are defined and discussed in Volume I of this series of reports. It is desired to calculate θ and possibly the angle of attack $\alpha = \theta - \gamma$ for "all values" of the true anomaly.

Even though the differential equation is a linear second order equation it does not lend itself readily to solution in terms of elementary or tabulated functions particularly since the variation of density with altitude is a tabulated function. Direct numeric integration appears to offer a rather straightforward approach, provided its limitation to a finite range of ν can be circumvented.

The means used to obtain results valid for all ν constitute the first section of this report. The second section describes the method of numeric integration adopted. The final section discusses the computer program written to implement these results while the program itself is given in the appendices.

1. DISCUSSION OF THE LINEAR SECOND ORDER DIFFERENTIAL EQUATION WITH PERIODIC COEFFICIENTS

It is the purpose of this section of the report to show that three integrations carried out over one orbital period suffice to furnish information about the boundedness or stability of solutions of Eq. (1). Equations with arbitrary periodic coefficients are discussed to insure independence of the tabulated atmospheric density function. In this form the presentation is an amplification of the concise discussion given by Coddington and Levinson, (Ref. 2) and a minor extension as well since the reference discusses homogeneous equations only.

1.1 General Case

Let

$$\ddot{y} + \alpha \dot{y} + \beta y = \gamma \tag{5}$$

be a linear differential equation in which α, β, and γ are periodic functions of t, with common period of 2π. The dots denote differentiation with respect to t. Eq. (1) is of this type. Let

$$\ddot{y} + \alpha \dot{y} + \beta y = 0 \tag{6}$$

be the associated homogeneous equation. Now let y_0 be that solution of Eq. (5) that has

$$y_0(0) = 0, \quad \dot{y}(0) = 0$$

and let y_1, y_2 be solutions of Eq. (6) that have respectively

$$y_1(0) = 1, \qquad \dot{y}_1(0) = 0,$$

$$y_2(0) = 0, \qquad \dot{y}_2(0) = 1.$$

Since these solutions are obviously linearly independent, any solution of Eq. (5) may be written

$$y = y_0 + a y_1 + b y_2 \;. \tag{7}$$

For $t = 0, 2\pi$ Eq. (7) yields

$$\left.\begin{aligned} y(0) &= y_0(0) + a y_1(0) + b y_2(0) = a \\ \dot{y}(0) &= \dot{y}_0(0) + a \dot{y}_1(0) + b \dot{y}_2(0) = b \end{aligned}\right\}$$

$$\left.\begin{aligned} y(2\pi) &= y_0(2\pi) + a y_1(2\pi) + b y_2(2\pi) \\ \dot{y}(2\pi) &= \dot{y}_0(2\pi) + a \dot{y}_1(2\pi) + b \dot{y}_2(2\pi) \end{aligned}\right\} \qquad (8)$$

If it is now required that y have period 2π,

$$y(2\pi) = y(0), \quad \dot{y}(2\pi) = \dot{y}(0) \qquad (9)$$

Eqs.(8) may be combined to yield

$$\left.\begin{aligned} y_0(2\pi) &= a[1 - y_1(2\pi)] - b y_2(2\pi) \\ \dot{y}_0(2\pi) &= -a\dot{y}_1(2\pi) + b[1 - \dot{y}_2(2\pi)] \end{aligned}\right\} \qquad (10)$$

Let

$$W = \begin{bmatrix} y_1(2\pi) & y_2(2\pi) \\ \dot{y}_1(2\pi) & \dot{y}_2(2\pi) \end{bmatrix}$$

then Eq. (10) may be written

$$\begin{bmatrix} y_0(2\pi) \\ \dot{y}_0(2\pi) \end{bmatrix} = (I - W)\begin{bmatrix} a \\ b \end{bmatrix}$$

A unique periodic solution to Eq. (5) is thus seen to exist if $(I - W)$ is non-singular. It will be shown below that

$$\det(I - W) = 0$$

implies a resonance in the conventional sense. In the non-resonant case let the periodic particular solution be denoted by the subscript p:

$$y_p = y_0 + a y_1 + b y_2$$

where a and b are the solution to the system of equations given by Eqs. (10).

Now define the notation

$$\bar{y} = \max_{0 \le t \le 2\pi} (|y|) . \qquad (11)$$

Then

$$\bar{y}_p \le \bar{y}_0 + |a| \bar{y}_1 + |b| \bar{y}_2 ,$$

and y_p exists and is bounded whenever y_0 , y_1 , y_2 , exist on the interval $[0,2\pi]$, and $det\,(I - W) \neq 0$.

Under these conditions, any solution of Eq. (5) may be written

$$y = y_p + a_0 y_1 + b_0 y_2 \qquad (12)$$

where a_0 and b_0 are initial condition constants. The only change from Eq. (7) is the choice of the particular solution. The notation

$$t_n = t - 2n\pi \qquad (13)$$

will be useful. In terms of the conditions at $t = 2n\pi$ Eq. (12) becomes

$$y(t) = y_p(t_n) + a_n y_1(t_n) + b_n y_2(t_n)$$

and in turn, when $t_n = 2\pi$,

$$y(2\pi + 2n\pi) = y_p(2\pi) + a_n y_1(2\pi) + b_n y_2(2\pi)$$

$$= y_p(0) + a_{n+1} y_1(0) + b_{n+1} y_2(0) = y_p(0) + a_{n+1} .$$

Since $y_p(2\pi) = y_p(0)$,

$$a_{n+1} = a_n y_1(2\pi) + b_n y_2(2\pi) .$$

Similarly

$$b_{n+1} = a_n \dot{y}_1(2\pi) + b_n \dot{y}_2(2\pi) .$$

Thus

$$\begin{bmatrix} a_{n+1} \\ b_{n+1} \end{bmatrix} = W \begin{bmatrix} a_n \\ b_n \end{bmatrix} = W^{n+1} \begin{bmatrix} a_0 \\ b_0 \end{bmatrix} .$$

The solutions of Eq. (5) remain bounded if W^n remains bounded for all n. This in turn is the case if the magnitudes of the eigenvalues of W are at most 1. In this case an upper bound on the amplitude of solutions is

$$\bar{y} \le \bar{y}_p + \sqrt{a_0^2 + b_0^2}\,(\bar{y}_1 + \bar{y}_2). \tag{14}$$

1.2 Resonant Case

If $(I - W)$ is singular, the system of equations

$$0 = (I - W) \begin{bmatrix} a \\ b \end{bmatrix} \tag{15}$$

has an infinity of non-trivial solutions. Let one of these be

$$a = a^*, \quad b = b^* \tag{16}$$

and let

$$y^* = a^* y_1 + b^* y_2 . \tag{17}$$

Then Eq. (15) may be rewritten

$$0 = (I - W) \begin{bmatrix} a^* \\ b^* \end{bmatrix}$$

or

$$I \begin{bmatrix} a^* \\ b^* \end{bmatrix} = W \begin{bmatrix} a^* \\ b^* \end{bmatrix}$$

whence

$$\begin{bmatrix} y_1(0) & y_2(0) \\ \dot{y}_1(0) & \dot{y}_2(0) \end{bmatrix} \begin{bmatrix} a^* \\ b^* \end{bmatrix} = \begin{bmatrix} y_1(2\pi) & y_2(2\pi) \\ \dot{y}_1(2\pi) & \dot{y}_2(2\pi) \end{bmatrix} \begin{bmatrix} a^* \\ b^* \end{bmatrix} ,$$

so that

$$\begin{bmatrix} y^*(0) \\ \dot{y}^*(0) \end{bmatrix} = \begin{bmatrix} y^*(2\pi) \\ \dot{y}^*(2\pi) \end{bmatrix} . \tag{18}$$

Thus y^* is a non-trivial solution of period 2π of the homogeneous equation, Eq. (6). Since the period of y^* matches that of the forcing term of Eq. (5) this meets the accepted notions of resonance.

Conversely, if a solution of the homogeneous Eq. (6) satisfies Eq. (18) the argument may be reversed to show that Eq. (15) is valid.

In the resonant case, the system of equations, Eqs. (10), may or may not have a solution. If it does, the analysis for the non-resonant case applies, except that y_p is no longer unique. If, however, the system of equations, Eqs. (10) is inconsistent, no periodic solution of Eq. (5) exists. In this case let

$$y = y_0 + a_0 y_1 + b_0 y_2 \tag{19}$$

be some solution of Eq. (5), where a_0 and b_0 are again initial condition constants. By suitable choice of the initial condition constants Eq. (19) may be written

$$y(t) = y_0(t_n) + a_n y_1(t_n) + b_n y_2(t_n) \tag{20}$$

where t_n is defined in Eq. (13). Now

$$\begin{aligned} y(2\pi + 2n\pi) &= y_0(2\pi) + a_n y_1(2\pi) + b_n y_2(2\pi) \\ &= y_0(0) + a_{n+1} y_1(0) + b_{n+1} y_2(0) = a_{n+1} , \end{aligned}$$

and

$$\begin{aligned} \dot{y}(2\pi + 2n\pi) &= \dot{y}_0(2\pi) + a_n \dot{y}_1(2\pi) + b_n \dot{y}_2(2\pi) \\ &= \dot{y}_0(0) + a_{n+1} \dot{y}_1(0) + b_{n+1} \dot{y}_2(0) = b_{n+1} . \end{aligned}$$

Thus

$$\begin{bmatrix} a_{n+1} \\ b_{n+1} \end{bmatrix} = \begin{bmatrix} y_0(2\pi) \\ \dot{y}_0(2\pi) \end{bmatrix} + W \begin{bmatrix} a_n \\ b_n \end{bmatrix} ,$$

and, proceeding recursively

$$\begin{bmatrix} a_{n+1} \\ b_{n+1} \end{bmatrix} = \begin{bmatrix} y_0\ (2\pi) \\ \dot{y}_0\ (2\pi) \end{bmatrix} + W\left(\begin{bmatrix} y_0\ (2\pi) \\ \dot{y}_0\ (2\pi) \end{bmatrix} + W\begin{bmatrix} a_{n-1} \\ b_{n-1} \end{bmatrix}\right)$$

$$= (I + W)\begin{bmatrix} y_0\ (2\pi) \\ \dot{y}_0\ (2\pi) \end{bmatrix} + W^2\begin{bmatrix} a_{n-1} \\ b_{n-1} \end{bmatrix} \qquad (21)$$

$$= \cdots = \left(\sum_{i=0}^{n} W^i\right)\begin{bmatrix} y_0\ (2\pi) \\ \dot{y}_0\ (2\pi) \end{bmatrix} + W^{n+1}\begin{bmatrix} a_0 \\ b_0 \end{bmatrix} .$$

Now the vector $\begin{bmatrix} a_{n+1} & b_{n+1} \end{bmatrix}$ can remain bounded as n increases only if the sum

$$S_n = \sum_{i=0}^{n-1} W^i \qquad (22)$$

remains bounded.

Let σ_{ni} be the eigenvalues of S_n and ω_i be those of W, where $i = 1, 2$. The eigenvalues of the matric polynomial, Eq. (22) have the relation

$$\begin{aligned} \sigma_{ni} &= (1 - \omega_i^{\,n})/(1 - \omega_i) \;, && \omega_i \neq 0, 1 \\ &= 0 \;, && \omega_i = 0 \\ &= n \;, && \omega_i = 1 \end{aligned}$$

Thus, the definition

$$w = \max\left(\,|\omega_1|\,,\ |\omega_2|\,\right) \qquad (23)$$

and relation

$$0 < w < 1 \qquad (24)$$

imply

$$|\sigma_{ni}| \rightarrow |1/(1-\omega_i)| < 1/(1-w) \quad \text{as} \quad n \rightarrow \infty .$$

But if the relation, Eq. (24), is satisfied the second term of Eq. (21) is also bounded, and the amplitude of y is limited to

$$|y| \le \frac{1}{1-w} \sqrt{y_0(2\pi)^2 + \dot{y}_0(2\pi)^2} + (\bar{y}_1 + \bar{y}_2)\sqrt{a_0^2 + b_0^2} \qquad (25)$$

where the bars are defined in Eq. (11).

The derivation of the bound, given by Eq. (25), did not use the fact of resonance, thus it is applicable even to the non-resonant case, although in the latter case the bound given by Eq. (14) will usually be sharper. (It should be borne in mind that the symbols a_0 and b_0 have different significances in the two expressions.)

1.3 Stability Criteria

The eigenvalues of the matrix W satisfy the equation

$$0 = \begin{vmatrix} \omega - y_1(2\pi) & -y_2(2\pi) \\ -\dot{y}_1(2\pi) & \omega - \dot{y}_2(2\pi) \end{vmatrix} = \omega^2 - [y_1(2\pi) + \dot{y}_2(2\pi)]\,\omega + \det W$$

so that

$$\omega = \frac{y_1(2\pi) + \dot{y}_2(2\pi)}{2} \pm \sqrt{\left[\frac{y_1(2\pi) + \dot{y}_2(2\pi)}{2}\right]^2 - \det W} \,. \qquad (26)$$

When the eigenvalues are complex, they are conjugates, so

$$\omega_1 \omega_2 = \left[\frac{y_1(2\pi) + \dot{y}_2(2\pi)}{2}\right]^2 + \det W - \left[\frac{y_1(2\pi) + \dot{y}_2(2\pi)}{2}\right]^2$$

$$= \det W ,$$

and thus

$$|\omega_1| = \sqrt{\det W} .$$

When the eigenvalues are real, let the one larger in absolute value be ω_1. Then

$$|\omega_1| \geq \left|\frac{y_1(2\pi) + \dot{y}_2(2\pi)}{2}\right| \geq \sqrt{\det W} \; .$$

For bounded motions, therefore, it is necessary that

$$\det W \leq 1.$$

It is to be noted that $\det W$ is the Wronskian of the system (y_1, y_2), evaluated at $t = 2\pi$. Abel's formula for the Wronskian is:

$$\mathcal{W}(x) = \mathcal{W}(0)\, e^{\int_0^x \alpha(t)\, dt},$$

Thus

$$\det W = \mathcal{W}(2\pi) = \begin{vmatrix} y_1(0) & y_2(0) \\ \dot{y}_1(0) & \dot{y}_2(0) \end{vmatrix} e^{\int_0^{2\pi} \alpha(t)\, dt} = e^{\int_0^{2\pi} \alpha(t)\, dt}.$$

Now if α is identically zero, or even if it has an average value of zero on the interval $(0, 2\pi)$,

$$\det W = 1.$$

Then, referring to Eq. (23),

$$w \geq 1.$$

It may be concluded, therefore, that in the absence of the damping term, the resonant solutions are unbounded unless Eqs. (10) happen to be dependent. The non-resonant solutions were shown above, Eq. (14), to be bounded, unless $w > 1$.

1.4 Short Term Boundedness

In the application of the results obtained above to Eq. (1) it must be recalled from Volume I, Appendix C of this series of reports, that the constants (or elements) describing the orbit are themselves subject to drift as the orbit decays. Thus the results are applicable to some large but finite number of periods. Under such a limitation an ultimately unbounded motion may still be acceptable if its rate of growth is small. For the nth period the bound expressed by Eq. (25) may be written:

$$\bar{y}(t_{n-1}) \le \frac{w^{n}-1}{w-1}\sqrt{y_0(2\pi)^2+\dot{y}_0(2\pi)^2} + w^{n-1}(\bar{y}_1+\bar{y}_2)\sqrt{y(0)^2+\dot{y}(0)^2}; \quad (27)$$

while in the non-resonant case the bound given by Eq. (14) becomes

$$\bar{y}(t_{n-1}) \le \bar{y}_p + w^{n-1}\sqrt{[y_p(0)-y(0)]^2+[\dot{y}_p(0)-\dot{y}(0)]^2}. \quad (28)$$

Now the total lifetime of the satellite may be divided into regions during each of which the parameters of the orbit may be considered constant, and during each of which the growth or decay of the pitch amplitude may be estimated by means of Eq. (27), or in a non-resonant case by Eq. (28).

1.5 A "Built-In" Check on the Accuracy of the Numeric Integration

Let Eq. (5) have a unique periodic solution y_p and let

$$y_p = y_e + y_o$$

where y_e is the even part of y_p and y_o is the odd part of y_p. Further, suppose that in Eq. (5) the functions α and γ are odd functions of t, and β is an even function of t. Then Eq. (5) becomes

$$(\ddot{y}_e + \alpha\dot{y}_e + \beta y_e) + (\ddot{y}_o + \alpha\dot{y}_o + \beta y_o) = \gamma.$$

The parity properties of functions guarantee that the first parenthesis on the left is an even function of t, while the second parenthesis is an odd function. Since the right member is an odd function, the first parenthesis must vanish. If there existed a non-trivial even function y_e the uniqueness of y_p would be destroyed. Thus

$$y_e \equiv 0,$$

and y_p is an odd function, so that $y_p(0) = 0$. Then in the solution of Eqs. (10):

$$a = 0,$$

and

$$y_p = y_0 + b y_1.$$

This situation arises in Eq. (1) when the damping in pitch parameter, Γ_ϕ, vanishes. In this case the size of a computed from Eqs. (10) becomes a measure of the accuracy of the numeric integration used to obtain the coefficients in these equations.

2. INTEGRATION PROCEDURE

The choice of integration procedure involves a balance between accuracy, speed, stability, and auxiliary "housekeeping" requirements. Gill, (Ref. 3) has derived a very elegant version of the Runge-Kutta fourth order procedure in which the housekeeping is minimized, in that only one storage cell need be assigned to each variable to be integrated. Milne (Ref. 4, Section 38) earlier gave a predictor-corrector method which obtains the same accuracy with half as many substitutions into the differential equation, but requires that a history of past values of the variables be kept. Milne's predictor formula requires a history of four previous values of the derivative, and if it is desired to double the step-size in the course of the integration a history of seven previous values must be available, along with a record of the time at which the step size was last doubled. The starting procedure for the predictor-corrector method is necessarily relatively involved. Since the corrector formula, Simpson's Rule, requires a history of only two previous values, it would be desirable to have a predictor formula with an equally short time-base, even at the cost of some accuracy and stability. Milne (Ref. 4, Sections 30 and 31) gives a procedure for deriving such formulas to arbitrary specifications and for estimating their accuracy. The procedure is here used to obtain the desired formula.

2.1 Derivation of Predictor Formula

In the identity:

$$y(t_0+h) - y(t_0-h) = \int_{t_0-h}^{t_0+h} \dot{y}\, dt \tag{29}$$

the assumption that the second term on the right is a linear combination of $y(t_0)$, $y(t_0-h)$, $h\dot{y}(t_0)$ and $h\dot{y}(t_0-h)$, may be written

$$\int_{t_0-h}^{t_0+h} \dot{y}\, dt = A\,y(t_0) + B\,y(t_0-h) + C\,h\,\dot{y}(t_0) + D\,h\,\dot{y}(t_0-h)\,. \tag{30}$$

The coefficients A, B, C, D may be determined by requiring that Eq. (30) be exact for the first four terms of the Taylor expansion of about . Thus, let

$$y(t) = a_0 + a_1(t-t_0) + a_2\frac{(t-t_0)^2}{2} + a_3\frac{(t-t_0)^3}{6} + \cdots \tag{31}$$

where

$$a_n = \frac{d^n y(t_0)}{dt^n}\,.$$

Substitution of Eq. (31) into Eq. (29) and Eq. (30) gives respectively

$$\int_{t_0-h}^{t_0+h} \dot{y}\, dt \cong 2a_1 h + \tfrac{1}{3} a_3 h^3 ,$$

$$\int_{t_0-h}^{t_0+h} \dot{y}\, dt \cong A a_0 + B\left(a_0 - a_1 h + \frac{a_2}{2} h^2 - \frac{a_3}{6} h^3\right)$$

$$+ C a_1 h + D\left(a_1 h - a_2 h^2 + \frac{a_3}{2} h^3\right) .$$

Equating the coefficients of a_0 , $a_1 a_2$, and a_3 respectively, yields:

$$\left.\begin{aligned} A + B &= 0 \\ C + D - B &= 2 \\ \tfrac{1}{2} B - D &= 0 \\ \tfrac{1}{2} D - \tfrac{1}{6} B &= \tfrac{1}{3} \end{aligned}\right\} .$$

The solution to this system is

$$A = -4 , \quad B = 4 , \quad C = 4 , \quad D = 2 .$$

Then Eq. (30) becomes

$$\int_{t_0-h}^{t_0+h} \dot{y}\, dt \cong -4y(t_0) + 4y(t_0-h) + 4h\dot{y}(t_0) + 2h\dot{y}(t_0-h) ,$$

which yields in Eq. (29):

$$y(t_0+h) \cong y(t_0-h) + 4\left\{y(t_0-h) - y(t_0) + \frac{h}{2}[\dot{y}(t_0-h) + 2\dot{y}(t_0)]\right\} \qquad (32)$$

$$= y_p(t_0+h)$$

where the subscript p stands for "predicted value". Its derivation insures that Eq. (32) is exact to third order terms.

The Taylor series, Eq. (31) may be made exact by including the residue term:

$$y(t) = a_0 + a_1(t-t_0) + a_2\frac{(t-t_0)^2}{2} + a_3\frac{(t-t_0)^3}{6} + \frac{d^4y(s)}{dt^4}\cdot\frac{(t-t_0)^4}{24} \qquad (33)$$

where s is on the interval $[t_0, t]$.

Now the error committed by the use of $y_p(t_0+h)$ in place of $y(t_0+h)$ may be obtained by subtracting the right member of Eq. (32) from its left member. If each term in this difference is expressed in terms of Eq. (33) or its first derivative, the coefficients of a_0, a_1, a_2, and a_3 will be zero because Eq. (32) is accurate to third order terms. The residue terms will remain:

$$y(t_0+h) - y_p(t_0+h) = \frac{h^4}{24}\left[\frac{d^4y(s_1)}{dt^4} - 5\frac{d^4y(s_2)}{dt^4} + 8\frac{d^4y(s_3)}{dt^4}\right],$$

where s_1 is between t_0 and t_0+h, while s_2 and s_3 are between t_0 and t_0-h. The three distinct values arise from the fact that s in Taylor's Theorem, Eq. (33), cannot be expected to be the same for the expansions of $y(t_0+h)$, $y(t_0-h)$, and $\dot{y}(t_0-h)$. However, Eq. (32) may be shown to satisfy Milne's Theorem 2, (Ref. 4, Section 31) which in effect permits the replacement of s_1, s_2, and s_3 by a single s between t_0-h and t_0+h. The error becomes

$$\frac{d^4y(s)}{dt^4}\cdot\frac{h^4}{6},$$

and Eq. (32) may be rewritten:

$$y(t_0+h) = y(t_0-h) + 4\{y(t_0-h) - y(t_0) + \frac{h}{2}[\dot{y}(t_0-h) + 2\dot{y}(t_0)]\} + \frac{d^4y}{dt^4}\cdot\frac{h^4}{6} \quad . \qquad (34)$$

2.2 <u>Corrector Formula</u>

Simpson's rule is

$$y(t_0+h) \cong y(t_0-h) + \frac{h}{3}[\dot{y}(t_0-h) + 4\dot{y}(t_0) + \dot{y}(t_0+h)] = y_s(t_0+h) \qquad (35)$$

where the subscript s denotes "calculated from Simpson's rule". An error analysis for Simpson's rule yields

$$y(t_0+h) = y(t_0-h) + \frac{h}{3}[\dot{y}(t_0-h) + 4\dot{y}(t_0) + \dot{y}(t_0+h)] - \frac{d^5y}{dt^5}\cdot\frac{h^5}{90} \quad . \qquad (36)$$

In the derivation of the error term in Eq. (35) the assumption was made that the other terms on the right-hand side are known precisely. If this assumption is replaced by the assumption that $\dot{y}(t_0+h)$ was itself computed by Eq. (32) the error term is larger, in fact

$$y(t_0+h)=y(t_0-h)+\frac{h}{3}[\dot{y}(t_0-h)+4\dot{y}(t_0)+\dot{y}_p(t_0+h)]+4\frac{d^5y}{dt^5}\cdot\frac{h^5}{90} \quad . \tag{37}$$

If, however, $\dot{y}_s(t_0+h)$ is used in place of $\dot{y}(t_0+h)$ the error term is about

$$\frac{6}{5}\cdot\frac{d^5y}{dt^5}\cdot\frac{h^5}{90} \quad .$$

2.3 Integration Scheme

The predictor-corrector scheme is then:

a) With $y=\theta'$ use Eq. (34) to predict $\theta'(v+\Delta v)$ from $\theta'(v)$, $\theta'(v-\Delta v)$, $\theta''(v)$, $\theta''(v-\Delta v)$.

b) With $y=\theta$ use Eq. (36) to predict $\theta(v+\Delta v)$ from $\theta(v-\Delta v)$, $\theta'(v-\Delta v)$, $\theta'(v)$ and the predicted $\theta'(v+\Delta v)$.

c) Use the predicted $\theta(v+\Delta v)$ and $\theta'(v+\Delta v)$ in Eq. (1) to predict $\theta''(v+\Delta v)$.

d) Use the predicted $\theta''(v+\Delta v)$ in Eq. (35) with $y=\theta'$ to calculate the corrected $\theta'(v+\Delta v)$.

e) Use the corrected $\theta'(v+\Delta v)$ in Eq. (35) with $y=\theta$ to calculate the corrected $\theta(v+\Delta v)$.

f) Use the corrected $\theta(v+\Delta v)$ and $\theta'(v+\Delta v)$ in Eq. (1) to calculate the corrected $\theta''(v+\Delta v)$.

The small size of the error term in Eq. (36) makes it unlikely that an iteration of steps d, e, f would improve the result, if the prediction of steps a, b, c is at all adequate. This adequacy may be tested by comparing the predicted and corrected values of $\theta''(v+\Delta v)$. Since the error estimates for the prediction exceed those for the correction the difference between predicted and corrected values is a pessimistic estimate of the error incurred at each step of the integration. Let $\epsilon(\Delta v)$ be the magnitude of the difference between the predicted and corrected values of $\theta''(v+\Delta v)$, and let $\bar{\epsilon}$ be the maximum tolerable difference. Then if

$$\epsilon(\Delta v)\leq\bar{\epsilon}/32$$

the error estimates for Eq. (35) suggest that

$$\epsilon(2\Delta v)\leq\bar{\epsilon}.$$

Thus a sufficiently small ϵ justifies a doubling of the step size. If, however

$$\epsilon(\Delta v) > \overline{\epsilon}$$

the integration must be restarted with a smaller value of Δv.

Starting, as well as restarting may be accomplished by Taylor's formulas

$$\theta'(v+\Delta v) = \theta'(v) + \theta''(v)\Delta v + \frac{d^3\theta(s)}{dv^3}\cdot\frac{(\Delta v)^2}{2} \quad , \qquad (38)$$

$$\theta(v+\Delta v) = \theta(v) + \theta'(v)\Delta v + \theta''(v)\frac{(\Delta v)^2}{2} + \frac{d^3\theta(s)}{dv^3}\cdot\frac{(\Delta v)^3}{6} \quad , \qquad (39)$$

where s is between v and $v+\Delta v$. The relatively low powers of Δv appearing in the error terms of Eq. (38) and (39) suggest that small values of Δv be used to retain accuracy. Since the predictor-corrector scheme makes it easy to double the step size there is but small penalty for setting the initial value of Δv too small.

Computational experiments have been conducted comparing the procedure presented above with Gill's version (Ref. 3) of the Runge-Kutta fourth order procedure and with the Kutta fourth order formula as quoted by Milne (Ref. 5). All three processes gave essentially the same accuracy for the same step size; as mentioned above, the present process requires half as many substitutions into the differential equation. It should, however, be noted that the relatively large coefficients appearing in Eq. (34) tend to favor the build up of "inherited" errors, so that the present method cannot be expected to be as stable as the Runge-Kutta process for differential equations with unstable solutions.

The potential build-up of inherited error is not of concern in the present study for three reasons. Firstly, the range of integration is relatively short so that inherited error cannot build up very far. Secondly, the most critical cases are the most unstable cases, but in these the demonstration of instability is obvious. Thirdly, application of the accuracy criterion developed in Section 1.5 to the computer results justifies, a posteriori, the assertion that the error build-up was nil to four decimal places.

3. COMPUTER PROGRAM

Program Descriptions of the various components of the Computer Program written to calculate solutions of Eq. (1) are included in Appendices A to F. Sample input and output data are in Appendix G. The program has been checked out and used at the North American Aviation, Inc. Columbus IBM 704 computing installation to generate the numerical results discussed in Volume I of this series of reports. Certain systems features of this program, such as the choice of input medium (magnetic tape or punch-cards), assignment of magnetic tape unit numbers, labelling of library routines, etc. may have to be varied if this program is to operate successfully at other installations or on other computing machines. It is believed that the information furnished is adequate to enable anyone familiar with the IBM Fortran Manuals (for instance Ref. 6 and 7), and the operating system of his installation to adapt the source language deck to that installation.

The program was written in two portions. The first portion integrates the differential equation, Eq. (1), and its homogeneous part, and records the results on a magnetic tape as well as on the output medium. The second portion reads the magnetic tape and produces punched cards suitable for automatic plotting. The portions are discussed separately below and a brief discussion of the atmosphere table look-up routine is also given. This section concludes with some remarks on a potential application of the computer program.

3.1 Integration Portion of the Program

All input data is read by means of a "sparse data" routine so that information unchanged from one case to the next need not be written or key-punched repetitively. The only item of input data modified by the program is the identification number used to identify records on the intermediate storage tape, which is automatically advanced from record to record. To further facilitate the preparation of input data the primary input variables are presented in lists; the program systematically uses each combination of list entries. All angles in input and output are expressed in degrees; however, inside the computer and on the intermediate storage tape they are carried in radian units.

In the integration of the differential equations care is taken to provide common abscissas from one integration to the next. This is accomplished by using a computing interval obtained from the print interval by successive halving, and by not doubling the step size (when doubling is permitted by the criteria of Section 2) if this would cause the end of the print interval to fall inside a computing interval. The accuracy of the integration is further insured by a test which aborts the calculation if the step size does not expand in the first three intervals. If the step size fails to expand, the accuracy of the starting formulas Eqs. (38), (39) is less than that implied by the predictor-corrector method. The calculation should be attempted again with a smaller minimum step size. The integration is also aborted if the magnitude of any of the θ's exceeds 10^7, such an event would imply a gross data error, or a machine failure. The number of computing steps taken during each print interval is printed out so that the step size used may be deduced, the total number of steps for each integration is printed out as an aid to future computing time estimates. As compiled for the IBM 704, the program will integrate about 8.5 steps per second.

3.2 Card-Punching Portion of the Program

This section may be combined with the first section in a single computer run, or it may be used separately if the intermediate storage tape is saved from run to run. The second option permits inspection and editing of the curves to be punched into cards. Since the on-line punching of cards is a relatively expensive process the card format was designed to admit up to fifteen curves per card deck, while retaining complete flexibility as to size and placement of the plots. This portion also reads its input data (specifying the editing information) with the "sparse data" routine. The only input item modified by the program is the card deck identification number which serves as a signal that all information to be punched has been accumulated. This number is reset to zero after the card deck has been punched to indicate that the store of information is again incomplete.

The punched-card output for automatic plotting assumes a plotting device capable of accepting one coordinate pair per card. Abscissa and ordinate may be placed in any of sixteen four digit fields, or the abscissa may be suppressed in favor of an abscissa punched for another curve. The abscissa field will contain the independent variable v . The ordinate may contain any function of the form

$$f = |\omega_p| \, \theta_0 + \omega_1 \, \theta_1 + \omega_2 \, \theta_2 + \omega_\gamma \, \gamma$$

where θ_0 is the particular solution of Eq. (1) corresponding to γ_0 of Section 1,

θ_1, θ_2 are the solutions of the homogeneous part of Eq. (2) corresponding to γ_1 , γ_2 of Section 1,

γ is defined in Eq. (2)

and the ω's are input data.

For $\omega_\gamma = 0$, $\omega_p = 1$, f will be a solution of the differential equation, Eq. (1). When $\omega_\gamma = -1$, f will be the angle of attack, α . When $\omega_p = -1$, the input values of ω_1 and ω_2 will be ignored, instead the program will use the precomputed values needed to make f periodic, of orbital period.

The card decks produced will contain, in addition to the abscissa-ordinate pairs, four terminating cards. The first two of these are calibration check cards which will position the plotting head to the lower left hand and upper right hand corners of the plot respectively, the third contains an 11-punch in Column 1 which may be used to stop the plotting device, and the fourth is completely blank, serving as a separator between decks. The body of the deck, and the first two terminating cards have a 0-punch in Column 1, sequence number in Columns 5-8, and a deck identification number in Columns 2-4. The four digit fields consisting of Columns 9-12, 13-16, etc to 69-72 are numbered F = 1, 2, ..., 16 respectively and are the data fields proper (Columns 73-80 are not readily accessible to the IBM 704 computer).

The tape file of data is searched completely for the record required by the input data, so that in principle these records could be demanded in any order. The search is most efficient however if the records are demanded in the order in which they are stored on the tape.

3.3 Atmospheric Properties Subroutine

The atmospheric properties routine used in the program is capable of simulating atmosphere tables based on the ideal gas law and on linear variations of temperature, gas constant, and specific heat ratio with geopotential altitude. In the present program the routine was used to obtain the density distribution of the 1959 ARDC (Minzner) Atmosphere, (Ref. 1), which was represented by use of the Molecular Scale Temperature in place of the usual temperature. The detail input data are given in Appendix G.

3.4 A Potential Application

In its present form the program can furnish information about the stability in pitch of a given satellite in a given orbit. If the detailed motion is not of interest, only the first portion of the program is required. For a design study, knowledge of the stability boundaries in various parameter planes of the differential equation, Eq. (1), would be desirable. These occur when the magnitude of the larger eigenvalue ω given by Eq. (26) passes through the value 1. With minor modifications the subroutine INTEG of the first part of the present computer program could be used as the basis of a program to search iteratively for these stability boundaries. Points of departure for these searches could be obtained from the eigenvalues of the sine problem to which the present problem reduces itself when the orbital eccentricity is zero.

A NOTE ON THE APPENDICES

Appendices A to E are presented in the format of a standard SHARE program description.

APPENDIX A

PROGRAM DESCRIPTION - PITCHING MOTION OF A SATELLITE
NORTH AMERICAN AVIATION, INC. COLUMBUS
ENGINEERING PROGRAM DESCRIPTION

1. Identification

   a. Pitching Motion of a Satellite, IM 091
   b. O. C. Juelich, December 1960
   c. Research Group - 350
   d. Fortran Source Deck is up to date.

2. Purpose

Numerically integrates one particular and two homogeneous solutions of the pitch-equation. Solutions are printed, and optionally punched into cards in scaled form suitable for automatic plotting.

3. Restrictions

   a. Uses Tape 5 as input tape
      Tape 6 as printer output tape
      Tape 2 for intermediate storage of results
      Tape 7 to store part of the program
   b. Operates in N.A.A. monitor for IBM 704 Fortran

4. Method

   a. The mathematical method is described in Report WADD TR 61-133 Vol. III, of which this description is a part.
   b. The program is designed to terminate calculations if the step size has not adequately expanded in the first three print intervals or if any of the solutions exceed 10 million degrees in amplitude.
   c. Punched Card output is described in Section 3.2 of this report.

5. Use

The program consists of two parts. The first part generates solutions of the differential equation and records these on magnetic tape. The second part retrieves selected solutions and scales them for automatic plotting. The two parts can be combined in one machine run or they can be used on separate occasions if the intermediate storage tape (tape 2) is preserved. The latter alternative permits inspection of the printed results before card-punching is undertaken.

6. Coding Information

   a. The first part consists of a main program and two subroutines labeled INTEG, DIFFC. In addition subroutine ATMØS (1F113) and Utility File subroutine FXRCH are used, as well as Library subroutines SQRT, CØS, SIN, CHAIN, and Fortran system routines.

b. The second part is Chain 1 on Tape 7. It consists of a main program and a subroutine labelled SCALE. In addition Utility File Subroutines FXRCH, NPSCL, WRITE, are used, along with Fortran system routines.

c. Descriptions of subroutines ATMØS, FXRCH, NPSCL, and WRITE are given separately.

7. Input Data

a. All input data to both parts of the program is read by the Uility File Routine FXRCH into the Fortran CØMMØN Data Region.

b. Part one requires the following data:

Location	Symbol	Designation, Description
7	dv_{min}	Minimum computing step size bound, degrees The minimum computing step size is the largest number of the form $dv_{print}/2^n$ (n integral) and $\le dv_{min}$
9	dv_{print}	Print Interval, degrees Should be a factor of 360°.
28	RECID	Record Identification Number or Control Number. If this number is negative or zero the remaining data set is ignored. If RECID is negative part II is loaded and given control. If RECID is zero tape 2 is rewound and computation ends. If RECID is positive it becomes the identification number of the data case, if it is omitted the value of RECID from the previous data case increased by 1. is used.
30	r_E	Radius of Earth, feet Used in Eq. (4), but not in subroutine ATMØS.
31	μ	Product of Universal gravitational constant and mass of earth, ft^3/sec^2 Used in Eq. (3).
32	TBL	Table used for routine ATMØS (1F113) See Appendix B.

Location	Symbol	Designation, Description
201	n_i	Number of orbit inclinations,
202	i_1	first orbit inclination, degrees
203	i_2	second orbit inclination
et seq.		
211	n_h	Number of perigee altitudes,
212	h_1	first perigee altitude, feet
213	h_2	second perigee altitude
et seq.		
221	n_ϵ	Number of eccentricities,
222	ϵ_1	first eccentricity, dimensionless
223	ϵ_2	second eccentricity
et seq.		
231	n_Γ	Number of pitching moment parameters,
232	Γ_1	first pitching moment parameter
233	Γ_2	second pitching moment parameter
et seq.		
241	n_{Γ_q}	Number of damping in pitch parameters
242	Γ_{q1}	first damping in pitch parameter
243	Γ_{q2}	second damping in pitch parameter
et seq.		
251	n_M	Number of inertia ratio parameters
252	M_1	first inertia ratio parameter
253	M_2	second inertia ratio parameter
et seq.		

The $n_i\, n_h\, n_\epsilon\, n_\Gamma\, n_{\Gamma_q} n_M$ combinations of the listed values of $i, h, \epsilon, \Gamma, \Gamma_q, M$ will be used systematically. The results will be identified by successive values of RECID.

c. Part two requires the following data:

Location	Symbol	Designation, Description
1.	RECID	Identification number of tape record to be plotted.
2.	w_p	Weight of function θ_0 in curve to be plotted Should be 1, 0, or -1.
3.	w_γ	Weight of $\gamma(v)$ in curve to be plotted Should be 0 or -1.
4.	w_1	Weight of function θ_1 in curve to be plotted Ignored when w_p = -1.
5.	w_2	Weight of function θ_2 in curve to be plotted Ignored when w_p = -1.

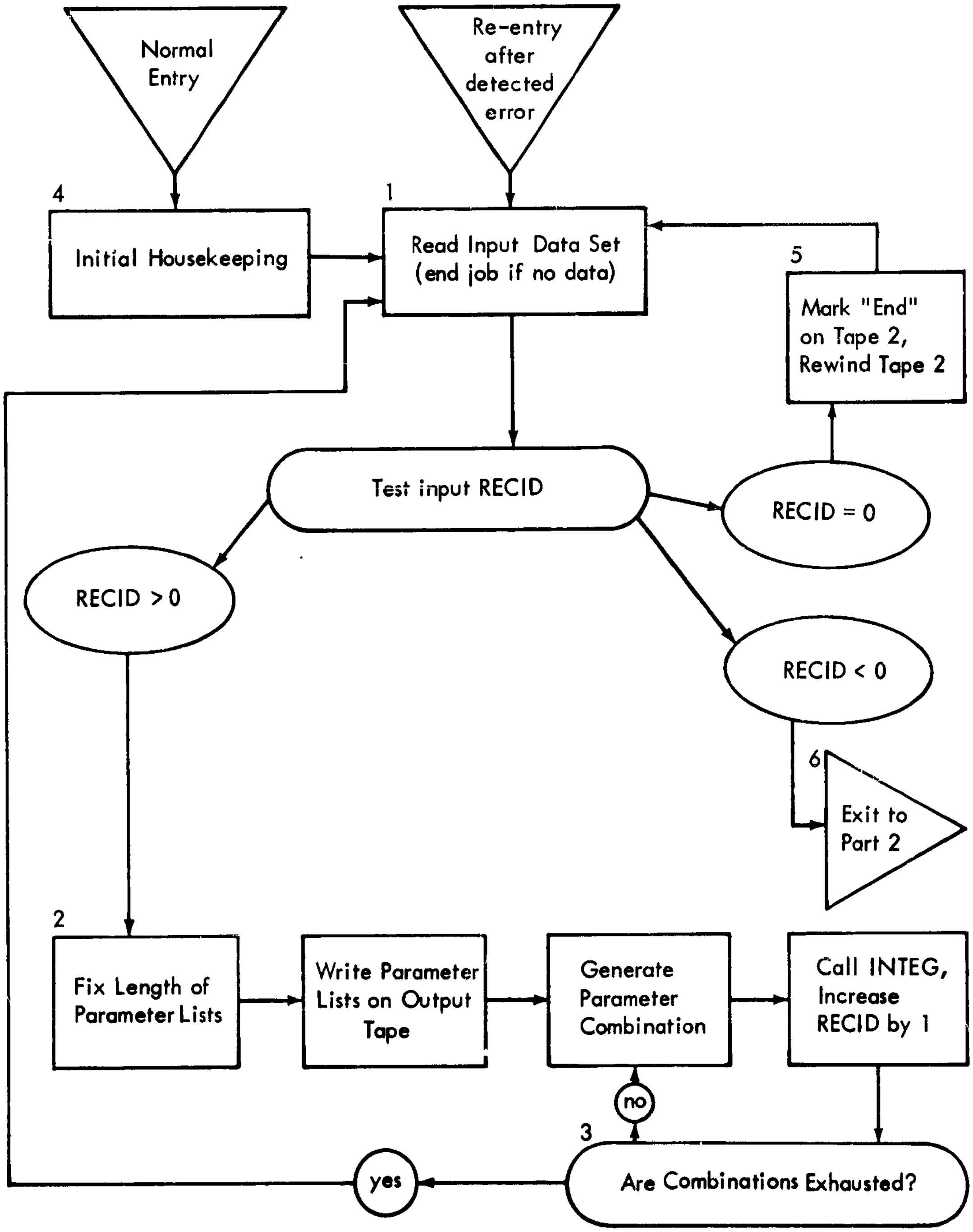
Location	Symbol	Designation, Description
6.	f_{min}	Minimum value of function in plotting interval, degrees If f is less than this value the point will be plotted on the bottom margin of the paper.
7.	c_{min}	Plotter counts for f_{min}.
8.	f_{max}	Maximum value of function in plotting interval, degrees If f exceeds this value the point will be plotted on the top margin of the paper.
9.	c_{max}	Plotter counts for f_{max}.
10.	c_{edge}	Plotter counts for top margin of the paper. The bottom margin is at zero counts.
11.	F_f	Number of the card field in which f is to appear. Should be between 1. and 16.
12.	ν_{min}	Minimum value of ν in plotting interval, degrees If ν is less than this value the point will be plotted on the left margin of the paper.
13.	d_{min}	Plotter counts for ν_{min}
14.	ν_{max}	Maximum value of ν in plotting interval, degrees. If ν exceeds this value the point will be plotted on the right margin of the paper.
15.	d_{max}	Plotter counts for ν_{max}
16.	d_{edge}	Plotter counts for right margin of the paper. The left margin is at zero counts.
17.	F_ν	Number of the card field in which ν is to appear, or control number. If F_ν is zero, ν will not be entered on the punched cards.
18.	CID	Identification number for card deck to be punched, or control number. If CID is zero the scaled data are accumulated in core storage for later punching.

Location	Symbol	Designation, Description
		If CID is not zero the accumulated information is punched into cards and CID becomes an identification number in columns 2 to 4 of each card. After punching CID is reset to zero.

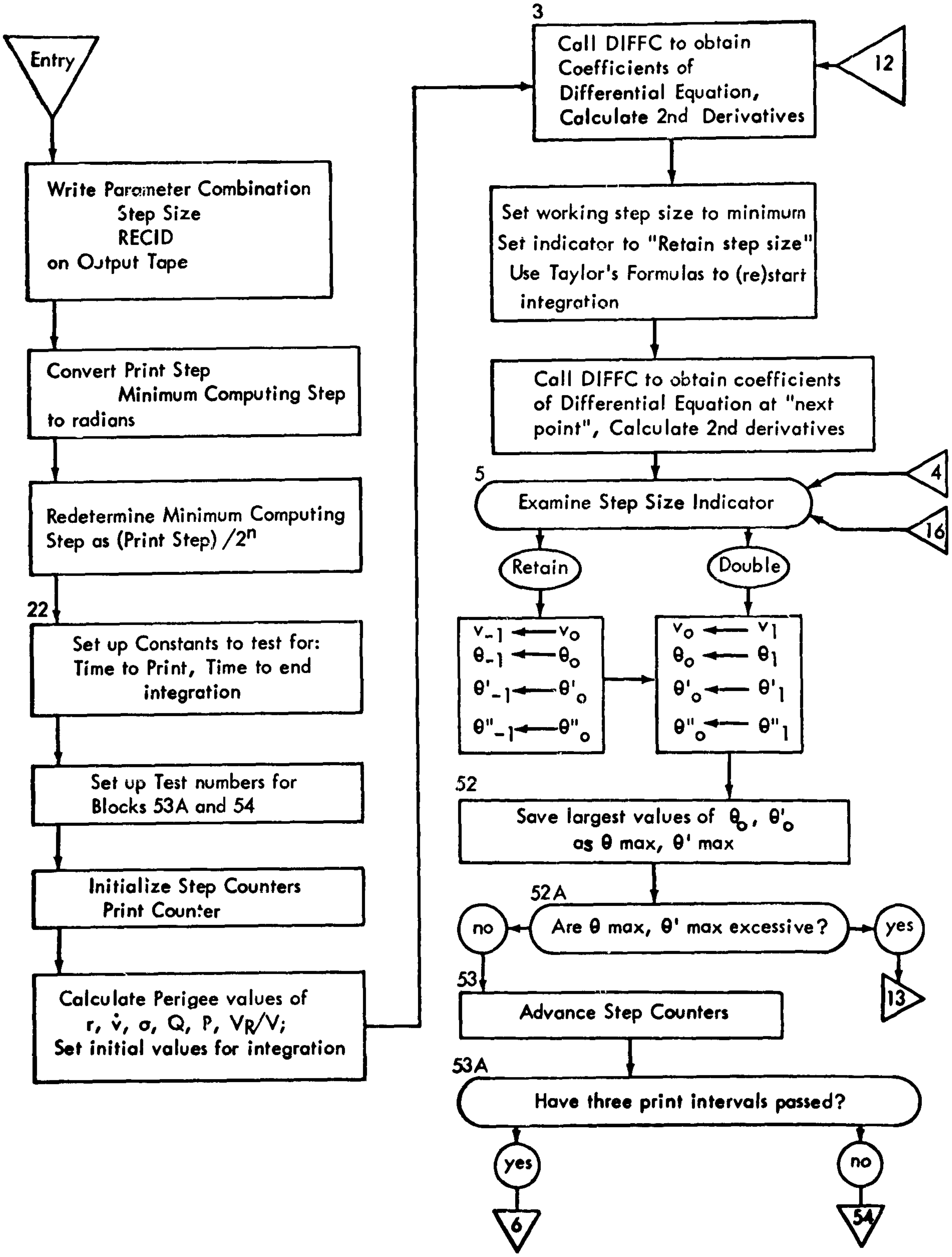
d. Sample Input Data with resulting output follow the compiled listings of the components of this program.

Note: The program symbol for v is PHI, for $\det(I-W)$ is DELTA.

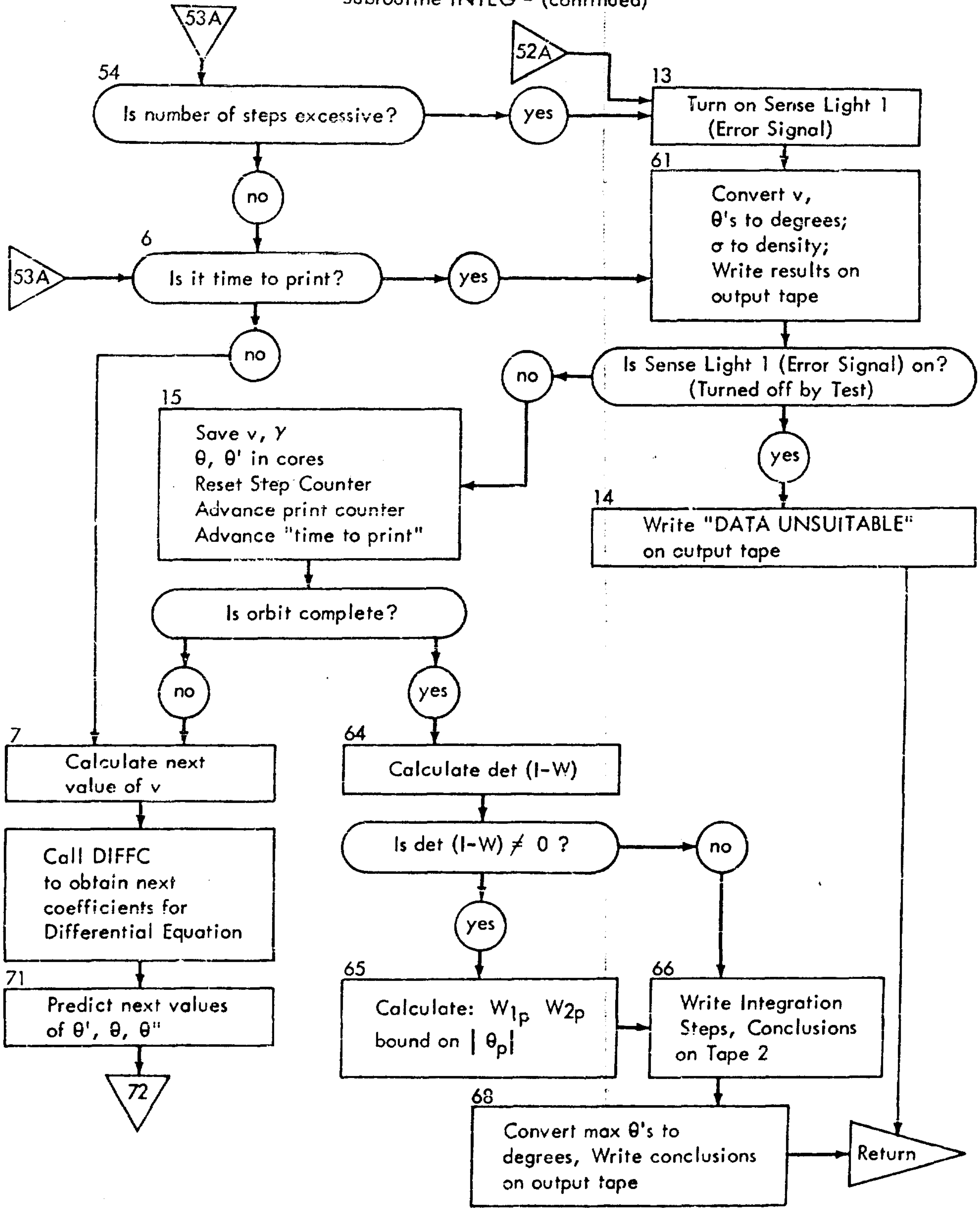
Main Program for Part I


Normal Entry
Re-entry after detected error
4
Initial Housekeeping
1
Read Input Data Set (end job if no data)
5
Mark "End" on Tape 2, Rewind Tape 2
Test input RECID
RECID = 0
RECID > 0
RECID < 0
6
Exit to Part 2
2
Fix Length of Parameter Lists
Write Parameter Lists on Output Tape
Generate Parameter Combination
Call INTEG, Increase RECID by 1
no
3
yes
Are Combinations Exhausted?

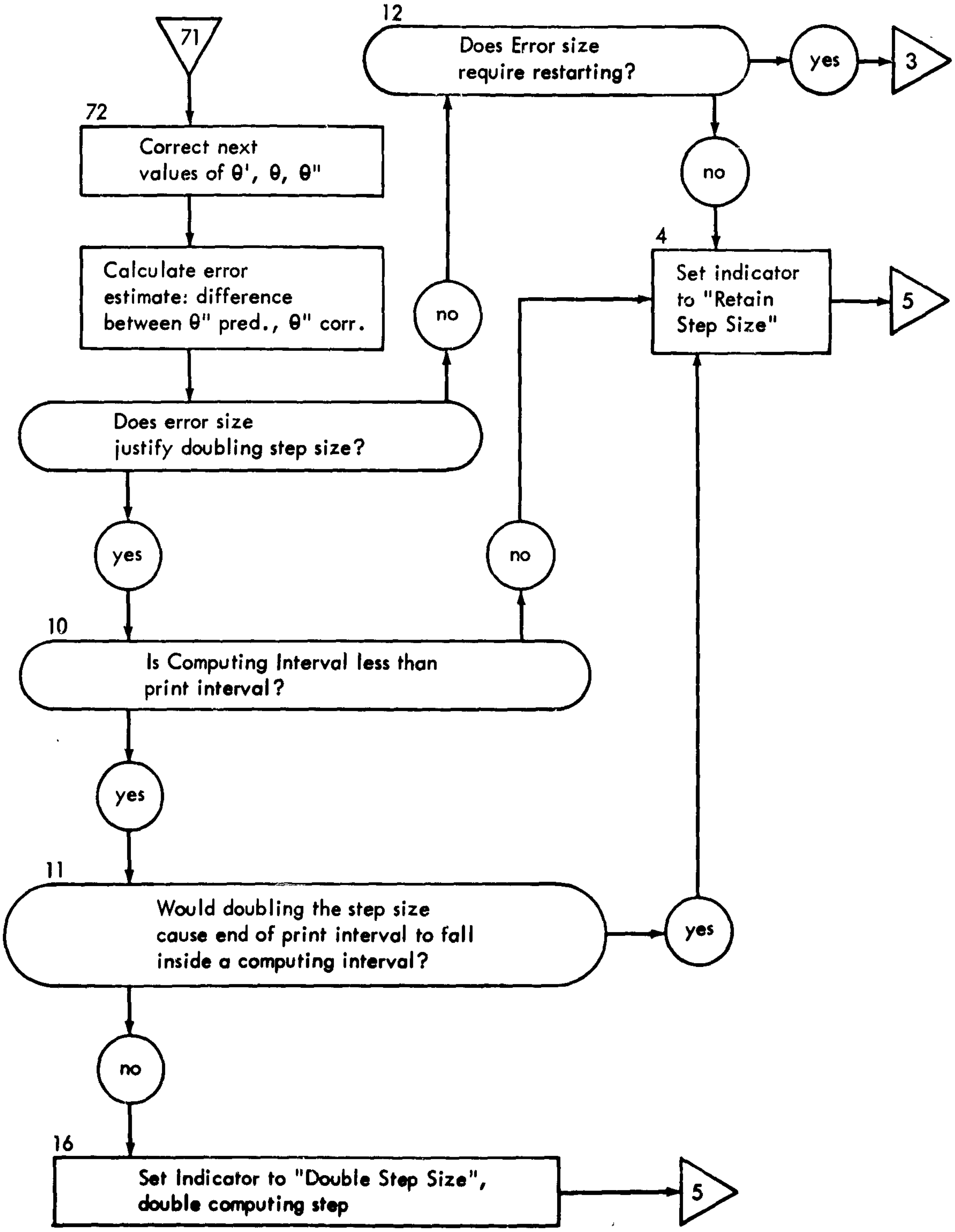
Subroutine INTEG


Entry
Write Parameter Combination Step Size RECID on Output Tape
Convert Print Step Minimum Computing Step to radians
Redetermine Minimum Computing Step as (Print Step) /2^n
22
Set up Constants to test for: Time to Print, Time to end integration
Set up Test numbers for Blocks 53A and 54
Initialize Step Counters Print Counter
Calculate Perigee values of r, v̇, σ, Q, P, VR/V; Set initial values for integration
3
Call DIFFC to obtain Coefficients of Differential Equation, Calculate 2nd Derivatives
12
Set working step size to minimum Set indicator to "Retain step size" Use Taylor's Formulas to (re)start integration
Call DIFFC to obtain coefficients of Differential Equation at "next point", Calculate 2nd derivatives
5
Examine Step Size Indicator
4
16
Retain
Double
v-1 ← vo
θ-1 ← θo
θ'-1 ← θ'o
θ"-1 ← θ"o
vo ← v1
θo ← θ1
θ'o ← θ'1
θ"o ← θ"1
52
Save largest values of θo, θ'o as θ max, θ' max
52A
Are θ max, θ' max excessive?
no
yes
13
53
Advance Step Counters
53A
Have three print intervals passed?
yes
no
6
54

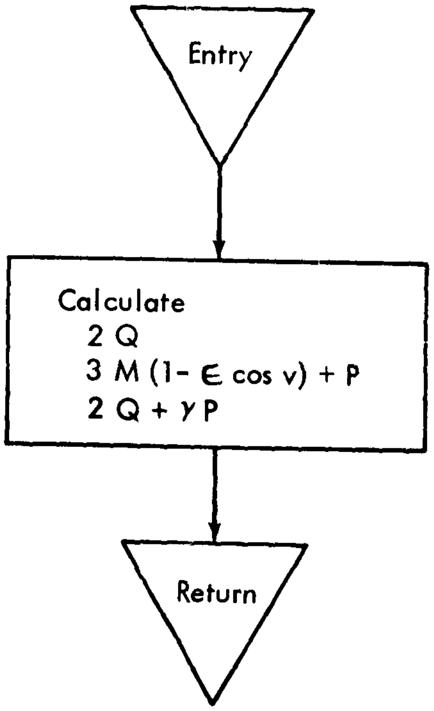
Subroutine INTEG - (continued)


53A
52A
54
Is number of steps excessive?
yes
13
Turn on Sense Light 1 (Error Signal)
no
61
Convert v, θ's to degrees; σ to density; Write results on output tape
6
53A
Is it time to print?
yes
no
Is Sense Light 1 (Error Signal) on? (Turned off by Test)
no
15
Save v, γ θ, θ' in cores Reset Step Counter Advance print counter Advance "time to print"
yes
14
Write "DATA UNSUITABLE" on output tape
Is orbit complete?
no
yes
7
Calculate next value of v
64
Calculate det (I-W)
Call DIFFC to obtain next coefficients for Differential Equation
Is det (I-W) ≠ 0 ?
no
yes
71
Predict next values of θ', θ, θ''
65
Calculate: W_{1p} W_{2p} bound on $|\theta_p|$
66
Write Integration Steps, Conclusions on Tape 2
72
68
Convert max θ's to degrees, Write conclusions on output tape
Return

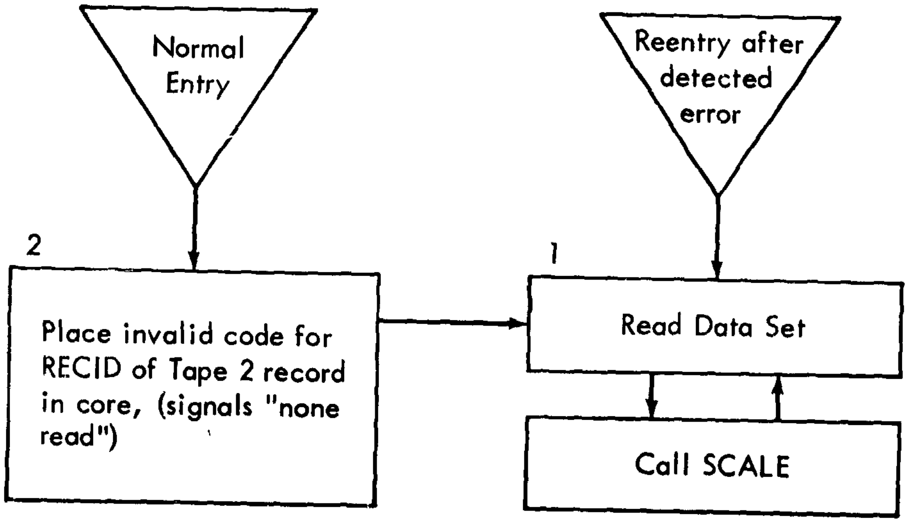
Subroutine INTEG - (concluded)


71
72
Correct next values of θ', θ, θ"
Calculate error estimate: difference between θ" pred., θ" corr.
Does error size justify doubling step size?
no
12
Does Error size require restarting?
yes
3
no
4
Set indicator to "Retain Step Size"
5
yes
no
10
Is Computing Interval less than print interval?
yes
11
Would doubling the step size cause end of print interval to fall inside a computing interval?
yes
no
16
Set Indicator to "Double Step Size", double computing step
5

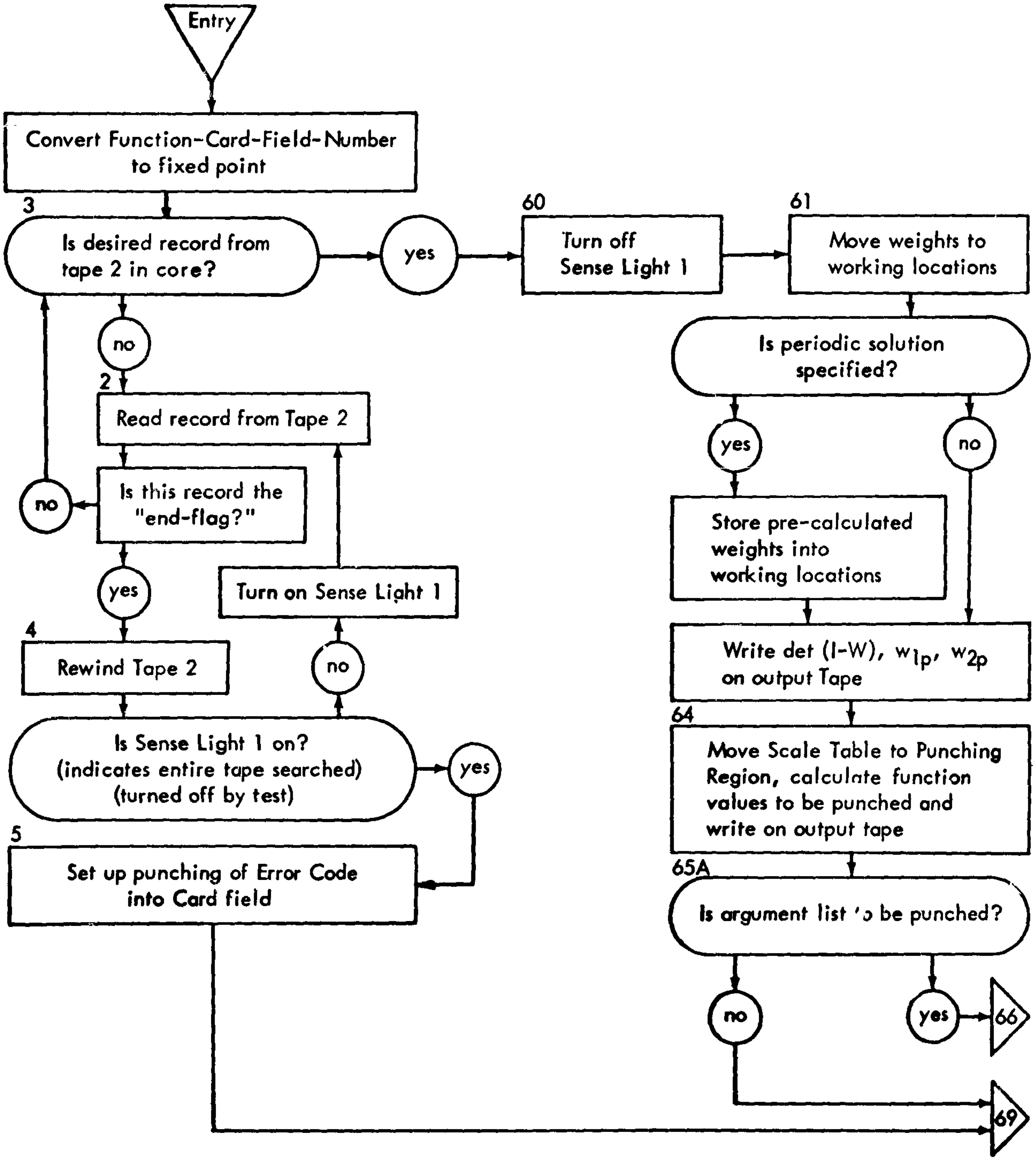
Subroutine DIFFC



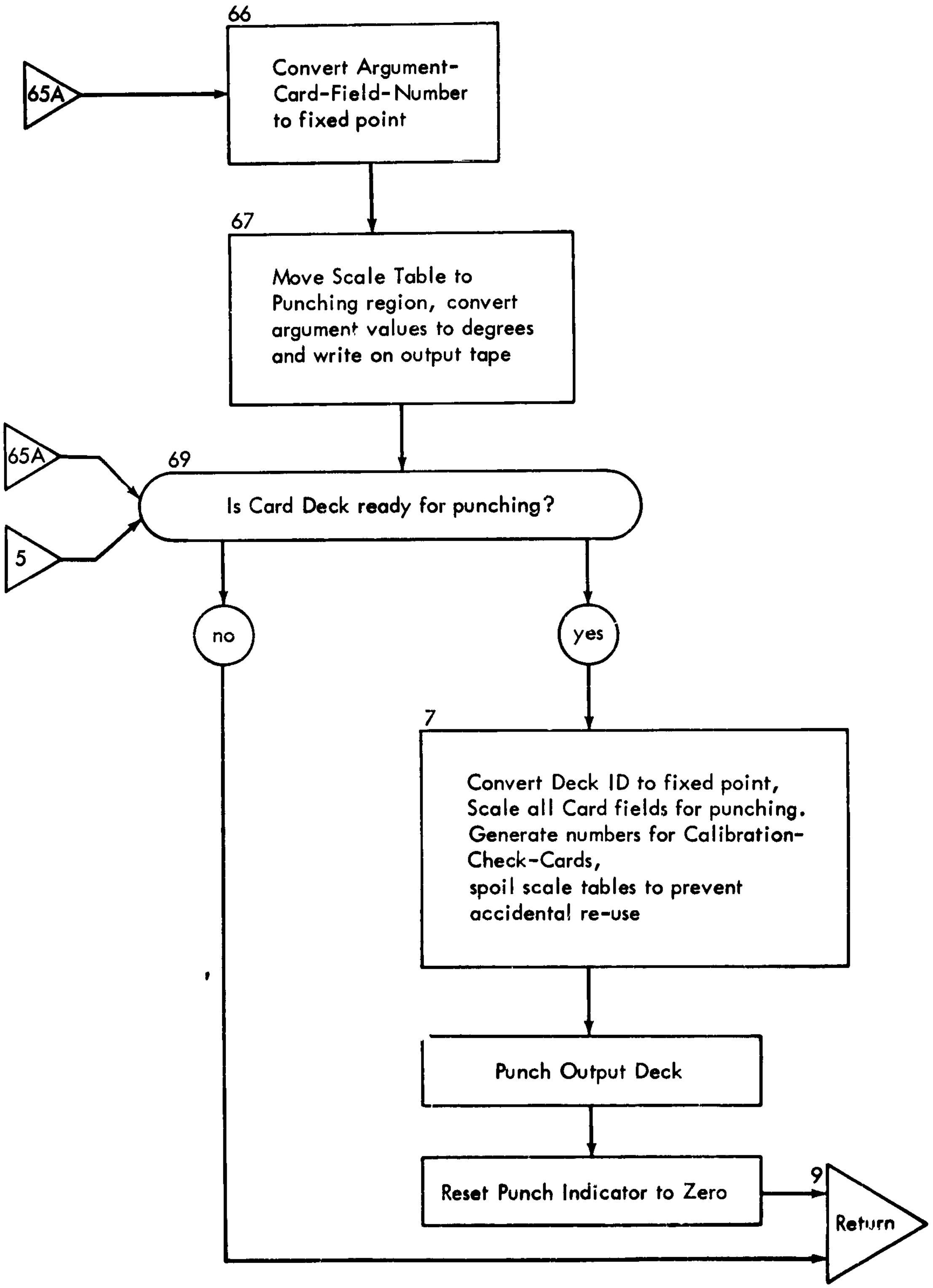
Main Program for Part II


Normal Entry
Reentry after detected error
2
Place invalid code for RECID of Tape 2 record in core, (signals "none read")
1
Read Data Set
Call SCALE

Subroutine SCALE


Entry
Convert Function-Card-Field-Number to fixed point
3
Is desired record from tape 2 in core?
yes
60
Turn off Sense Light 1
61
Move weights to working locations
no
2
Read record from Tape 2
Is this record the "end-flag?"
no
yes
Turn on Sense Light 1
4
Rewind Tape 2
no
Is Sense Light 1 on? (indicates entire tape searched) (turned off by test)
yes
5
Set up punching of Error Code into Card field
Is periodic solution specified?
yes
no
Store pre-calculated weights into working locations
Write det (I-W), w_{1p}, w_{2p} on output Tape
64
Move Scale Table to Punching Region, calculnte function values to be punched and write on output tape
65A
Is argument list 'o be punched?
no
yes
66
69

Subroutine SCALE - (concluded)


66
65A
Convert Argument-Card-Field-Number to fixed point
67
Move Scale Table to Punching region, convert argument values to degrees and write on output tape
65A
69
Is Card Deck ready for punching?
5
no
yes
7
Convert Deck ID to fixed point, Scale all Card fields for punching. Generate numbers for Calibration-Check-Cards, spoil scale tables to prevent accidental re-use
Punch Output Deck
Reset Punch Indicator to Zero
9
Return

```
C     MAIN PROGRAM TO VARY THROUGH LISTS OF VARIABLES                   1M091300
      DIMENSION C(260),D(260)                                           1M091302
      COMMON C                                                          1M091304
      EQUIVALENCE (C,D)                                                 1M091306
      IF DIVIDE CHECK 1,4                                               1M091307
    4 REWIND 2                                                          1M091308
      YD=-1.                                                            1M091309
      NS=1                                                              1M091310
    1 CALL FXRCH(D)                                                     1M091312
      IF (C(28) ) 6,5,2                                                 1M091314
    2 CONTINUE                                                          1M091316
      IM=D(201)                                                         1M091318
      JM=D(211)                                                         1M091320
      KM=D(221)                                                         1M091322
      LM=D(231)                                                         1M091324
      MM=D(241)                                                         1M091326
      NM=D(251)                                                         1M091328
      WRITE OUTPUT TAPE 6,11,(D(I+201),I=1,IM)                          1M091330
      WRITE OUTPUT TAPE 6,12,(D(I+211),I=1,JM)                          1M091332
      WRITE OUTPUT TAPE 6,13,(D(I+221),I=1,KM)                          1M091334
      WRITE OUTPUT TAPE 6,14,(D(I+231),I=1,LM)                          1M091336
      WRITE OUTPUT TAPE 6,15,(D(I+241),I=1,MM)                          1M091338
      WRITE OUTPUT TAPE 6,16,(D(I+251),I=1,NM)                          1M091340
      DO 3 I=1,IM                                                       1M091342
      DO 3 J=1,JM                                                       1M091344
      DO 3 K=1,KM                                                       1M091346
      DO 3 L=1,LM                                                       1M091348
      DO 3 M=1,MM                                                       1M091350
      DO 3 N=1,NM                                                       1M091352
      C(1)=D(I+201)                                                     1M091354
      C(2)=D(J+211)                                                     1M091356
      C(3)=D(K+221)                                                     1M091358
      C(4)=D(L+231)                                                     1M091360
      C(5)=D(M+241)                                                     1M091362
      C(6)=D(N+251)                                                     1M091364
      CALL INTEG                                                        1M091366
      C(28)=C(28)+1.                                                    1M091367
    3 CONTINUE                                                          1M091368
      GO TO 1                                                           1M091370
   11 FORMAT (18H1 PARAMETER LIST -/7H0  OI =(9F10.2))                  1M091372
   12 FORMAT (7H0  HP =(9F10.0))                                        1M091374
   13 FORMAT (7H0  EC =(9F10.6))                                        1M091376
```

```
 14    FORMAT (7HO GAM =(9F10.3))                                    1M091378
 15    FORMAT (7HOGAMQ =(9F10.3))                                    1M091380
 16    FORMAT (7HO  OM =(9F10.6))                                    1M091382
 5     WRITE TAPE 2,YD,NS,(C(I),I=1,15)                              1M091384
       END FILE 2                                                    1M091386
       REWIND 2                                                      1M091388
       GO TO 1                                                       1M091390
 6     C(8)=CHAINF(1,7)                                              1M091392
       STOP                                                          1M091394
       END(0,0,0,0,0)
```

```
        STORAGE FOR VARIABLES APPEARING IN COMMON SENTENCES

               DEC    OCT                DEC    OCT                DEC    OCT                DEC    OCT
             D 32562 77462             C 32562 77462

           EXTERNAL FORMULA NUMBERS WITH CORRESPONDING INTERNAL FORMULA NUMBERS AND OCTAL LOCATIONS

       EFN    IFN   LOC           EFN    IFN   LOC           EFN    IFN   LOC           EFN    IFN   LOC
         4      5 00012             4      5 00012             1      8 00017             1      9 00017
         2     11 00027             3     68 00343            11     70 00000            12     71 00000
        13     72 00000            14     73 00000            15     74 00000            16     75 00000
         5     76 00366             5     82 00400             6     86 00405

                                         STORAGE NOT USED BY PROGRAM

               DEC    OCT                DEC    OCT
               317 00475               32302 77056

                                      LOCATIONS OF NAMES IN TRANSFER VECTOR

               DEC    OCT                DEC    OCT                DEC    OCT                DEC    OCT
       INTEG     2 00002      FXRCH        7 00007      CHAIN        0 00000      (FIL)        3 00003
      (IOH)O     5 00005      (LEV)        6 00006      (RTN)        1 00001      (STH)        4 00004

         STORAGE LOCATIONS FOR VARIABLES NOT APPEARING IN DIMENSION,EQUIVALENCE OR COMMON SENTENCES

               DEC    OCT                DEC    OCT                DEC    OCT                DEC    OCT
          IM   316 00474          JM     315 00473          KM     314 00472          LM     313 00471
          MM   312 00470          NM     311 00467          NS     310 00466          YD     309 00465

                        STORAGE LOCATIONS FOR SYMBOLS NOT APPEARING IN SOURCE PROGRAM

               DEC    OCT                DEC    OCT                DEC    OCT                DEC    OCT
     E)10K     198 00306    E)10J      191 00277    D)20L      203 00313    D)20K      199 00307
     D)20J     192 00300    D)20I      185 00271    C)G5       308 00464    C)G4       307 00463
     C)G3      306 00462    C)G2       305 00461    C)G1       304 00460    8)G        280 00430
     8)F       284 00434    8)E        288 00440    8)D        292 00444    8)C        296 00450
     8)B       303 00457    2)         269 00415    3)         271 00417    6)         272 00420

                                    SUBROUTINES NOT PUNCHED FROM LIBRARY
      FXRCH          (LEV)        (IOH)O        (STH)        (FIL)        INTEG        (RTN)        CHAIN
      SUBROUTINE INTEG                                                  1M091416
      PREDF(YL,YO,YPL,YPO)=YL+4.*(YL-YO+DPC*(YPL+YPO                    1M091418
     2 +YPO)/2.)                                                        1M091420
      SIMPF(YL,YPL,YPO,YPN)=YL+DPC*(YPL+4.*YPO+YPN)/3.                  1M091422
      RELERF(A,B)=ABSF(A-B)/MAX1F(ABSF(A),ABSF(B),1.E-7)                1MG91424
      DIMENSION C(200),VMAX(2,3),PHI(3),V(3,4,3),CO(3),                 1M091426
     2 SAV(2,4,360),PRBL(8)                                             1M091427
      COMMON C                                                          1M091428
      EQUIVALENCE (C(32),TBL(1),SIG(14),R(15),H(16),QP(17),             1M091430
     2 PP(18),VRVV(19),SIGP(20),THPC(21),THO(22),PHIO(23),              1M091432
     3 EC(30),HP(31),OI(32),PHMAX(5),OM(27),GAMQ(28),GAM(29))           1M091434
     4 ,(GAMMA(4),RE(3),OMU(2),C(32) ),(N,PRBL(8) )                     1M091436
      WRITE OUTPUT TAPE 6,1,(C(I),I=1,7),C(9),C(28)                     1M091438
1     FORMAT (1H18X2HOI13X2HHP13X2HEC13X3HGAM11X4HGAMQ12X                1M091440
     2 2HOM/F13.2,F15.0,F15.6,1PE17.4,E15.4,0PF13.6/1H0                 1M091442
     3 4X9HD PHI MIN5X11HD PHI PRINT51X9HRECORD ID/                     1M091444
     4 1P2E15.4,0PF60.0/1H08X3HPHI12X1HH7X3HRHO16X5HGAMMA               1M091446
     5 6X8HALPHA H16X8HALPHA H2 6X8HALPHA PA6X1HN)                      1M091448
      DPPR=C(9)/57.295780                                               1M091450
      DPLIM=C(7)/57.295780                                              1M091452
      DPMIN=DPPR                                                        1M091454
23    DPTST=DPMIN/2.                                                    1M091456
      IF (DPMIN-DPLIM) 22,22,21                                         1M091458
21    DPMIN=DPMIN/2.                                                    1M091460
      GO TO 23                                                          1M091462
22    PHPR=DPPR-DPTST                                                   1M091464
      PHEND=6.2831853-DPTST                                             1M091466
      FNRAT=DPPR/DPTST                                                  1M091468
      NRAT=FNRAT                                                        1M091470
      N=0                                                               1M091472
      NT=0                                                              1M091474
      NS=1                                                              1M091476
      RP=HP+RE                                                          1M091478
      PHDP=SQRTF(OMU*(1.+EC)/RP)/RP                                     1M091480
      CALL ATMOS (HP,TBL,R)                                             1M091482
      SIGP=SIG                                                          1M091484
      QP=GAMQ*SIGP*RP*2.9711463E-4                                      1M091486
      PP=GAM*SIGP*RP**2*1.1884585E-3                                    1M091488
      VRVV=1.-7.292 1151E-5*COSF(OI/57.295780)/PHDP                     1M091490
      PHI(2)=0.                                                         1M091492
      V(1,2,1)=1.7453293E-2                                             1M091494
      VMAX(1,1)=1.7453293E-2                                            1M091496
      V(1,2,2)=0.                                                       1M091498
      VMAX(1,2)=0.                                                      1M091500
      V(1,2,3)=0.                                                       1M091502
      VMAX(1,3)=0.                                                      1M091504
      V(2,2,1)=0.                                                       1M091506
      VMAX(2,1)=0.                                                      1M091508
      V(2,2,2)=1.                                                       1M091510
      VMAX(2,2)=1.                                                      1M091512
      V(2,2,3)=0.                                                       1M091514
      VMAX(2,3)=0.                                                      1M091516
3     CALL DIFFC(PHI(2),CO)                                             1M091518
      DO 31 I=1,3                                                       1M091520
31    V(3,2,I)=-V(2,2,I)*CO(1)-V(1,2,I)*CO(2)                           1M091522
      V(3,2,3)=V(3,2,3)+CO(3)                                           1M091524
      DPC=DPMIN                                                         1M091526
      M=1                                                               1M091528
      PHI(3)=PHI(2)+DPC                                                 1M091530
      CALL DIFFC(PHI(3),CO)                                             1M091532
      DO 32 I=1,3                                                       1M091534
      V(1,3,I)=V(1,2,I)+DPMIN*(V(2,2,I)+DPMIN*V(3,2,I)/2.)              1M091536
      V(2,3,I)=V(2,2,I)+DPMIN*V(3,2,I)                                  1M091538
32    V(3,3,I)=-V(2,3,I)*CO(1)-V(1,3,I)*CO(2)                           1M091540
      V(3,3,3)=V(3,3,3)+CO(3)                                           1M091542
5     DO 51 J=M,2                                                       1M091544
      PHI(J)=PHI(J+1)                                                   1M091546
      DO 51 K=1,3                                                       1M091548
      DO 51 I=1,3                                                       1M091550
51    V(I,J,K)=V(I,J+1,K)                                               1M091552
      DO 52 J=1,3                                                       1M091554
      DO 52 I=1,2                                                       1M091556
52    VMAX(I,J)=MAX1F(VMAX(I,J),ABSF(V(I,2,J) )  )                      1M091558
      IF (MAX1F(VMAX(1,1),VMAX(1,2),VMAX(1,3) )-174532.93) 53,13,13     1M091560
53    N=N+1                                                             1M091562
      NT=NT+1                                                           1M091564
      IF (PHI(2)-3.*DPPR) 54,6,6                                        1M091566
54    IF (NT-NRAT) 6,6,13                                               1M091568
6     IF (PHI(2)-PHPR) 7,61,61                                          1M091570
61    PRBL(1)=57.295780*PHI(2)                                          1M091572
      PRBL(2)=H                                                         1M091574
      PRBL(4)=57.295780*GAMMA                                           1M091576
      PRBL(3)=2.376 917E-3*SIG                                          1M091578
      DO 62 I=1,3                                                       1M091580
```

```
   62  PRBL(I+4)=57.295780*V(1,2,I)-FRBL(4)                        1M091582
       WRITE OUTPUT TAPE 6,2,(PRBL(I),I=1,8)                       1M091584
    2  FORMAT (F14.3,F14.0,1PE14.4,0P4F14.3,I7)                    1M091586
       IF (SENSELIGHT 1) 14,15                                     1M091588
   14  WRITE OUTPUT TAPE 6,17                                      1M091590
   17  FORMAT (16H0DATA UNSUITABLE)                                1M091592
       GO TO 9                                                     1M091594
   15  SAV(1,1,NS)=PHI(2)                                          1M091596
       SAV(2,1,NS)=GAMMA                                           1M091598
       DO 63 J=1,3                                                 1M091600
       DO 63 I=1,2                                                 1M091602
   63  SAV(I,J+1,NS)=V(I,2,J)                                      1M091604
       N=0                                                         1M091606
       NS=NS+1                                                     1M091608
       PHPR=PHPR+DPPR                                              1M091610
       IF (PHI(2)-PHEND) 7,7,64                                    1M091612
   64  PRBL(1)=(1.-57.295780*V(1,2,1) )*(1.-V(2,2,2) )             1M091614
      2 -V(2,2,1)*V(1,2,2)/1.7453293E-2                            1M091616
       PRBL(2)=0.                                                  1M091618
       PRBL(3)=0.                                                  1M091620
       PRBL(4)=0.                                                  1M091622
       IF (ABSF(PRBL(1) )-1.E-6) 66,66,65                          1M091624
   65  PRBL(2)=57.295780*(V(1,2,3)*(1.-V(2,2,2) )+V(2,2,3)         1M091626
      2 *V(1,2,2) )/PRBL(1)                                        1M091628
       PRBL(3)=( (1.-57.295780*V(1,2,1) )*V(2,2,3)+V(2,2,1)        1M091630
      2 *V(1,2,3)/1.7453293E-2 )/PRBL(1)                           1M091632
       PRBL(4)=VMAX(1,3)+ABSF(PRBL(2) )*VMAX(1,1)                  1M091634
      2 +ABSF(PRBL(3) )*VMAX(1,2)                                  1M091636
   66  DO 67 I=1,3                                                 1M091638
   67  PRBL(I+4)=VMAX(1,I)                                         1M091640
       NS8=8*NS-8                                                  1M091642
 C                                                                 1M091643
       DIMENSION SAV1(2880)                                        1M091644
       EQUIVALENCE (SAV1,SAV)                                      1M091646
 C                                                                 1M091647
       WRITE TAPE 2,C(28),NS8,(SAV1(I),I=1,NS8),(PRBL(I),I=1,7)    1M091648
       DO 68 I=1,4                                                 1M091650
   68  PRBL(I+3)=57.295780*PRBL(I+3)                               1M091652
       WRITE OUTPUT TAPE 6,1,(C(I),I=1,7),C(9)                     1M091654
       WRITE OUTPUT TAPE 6,69,C(28),FRBL(1),(V(2,2,I),I=1,3),      1M091656
      2 NT,PRBL(4),PRBL(2),(PRBL(I+4),I=1,3),PRBL(3),(VMAX(2,I),I=1,3) 1M091658
   69  FORMAT (1H+F89.0/1H031X5HDELTA23X15HEND DERIVATIVES28X2HNT/ 1M091660
    2 1PE42.4,0P14X3F14.3,I7//20X27HTH PER BD   WEIGHT THETA H1           1M091662
    3 13X10HMAX THETAS/18X1P2E12.4,14X0P3F14.3//32X                       1M091664
    4 15HWEIGHT THETA H213X15HMAX DERIVATIVES/1PE42.4,14X                 1M091666
    5 0P3F14.3)                                                           1M091668
9     RETURN                                                              1M091670
7     PHI(3)=PHI(2)+DPC                                                   1M091672
      CALL DIFFC(PHI(3),CO)                                               1M091674
      DO 71 I=1,3                                                         1M091676
      V(2,4,I)=PREDF(V(2,1,I),V(2,2,I),V(3,1,I),V(3,2,I) )                1M091678
      V(1,4,I)=SIMPF(V(1,1,I),V(2,1,I),V(2,2,I),V(2,4,I) )                1M091680
71    V(3,4,I)=-V(2,4,I)*CO(1)-V(1,4,I)*CO(2)                             1M091682
      V(3,4,3)=V(3,4,3)+CO(3)                                             1M091684
      DO 72 I=1,3                                                         1M091686
      V(2,3,I)=SIMPF(V(2,1,I),V(3,1,I),V(3,2,I),V(3,4,I) )                1M091688
      V(1,3,I)=SIMPF(V(1,1,I),V(2,1,I),V(2,2,I),V(2,3,I) )                1M091690
72    V(3,3,I)=-V(2,3,I)*CO(1)-V(1,3,I)*CO(2)                             1M091692
      V(3,3,3)=V(3,3,3)+CO(3)                                             1M091694
      ERR=0.                                                              1M091696
      DO 73 I=1,3                                                         1M091698
73    ERR=MAX1F(ERR,RELERF(V(3,4,I),V(3,3,I) ) )                          1M091700
      IF (ERR-1.E-6) 10,10,12                                             1M091702
10    IF (DPC+DPC-DPPR) 11,11,4                                           1M091704
11    TEST=.5*(PHPR+DPTST-PHI(3) )/DPC                                    1M091706
      IF (INTF((TEST-INTF(TEST+.5) )*FNRAT) ) 4,16,4                      1M091708
16    M=2                                                                 1M091710
      DPC=DPC+DPC                                                         1M091711
      GO TO 5                                                             1M091712
4     M=1                                                                 1M091714
      GO TO 5                                                             1M091716
12    IF (ERR-1.E-4) 4,4,3                                                1M091718
13    SENSELIGHT 1                                                        1M091720
      GO TO 61                                                            1M091722
      END(0,0,0,0,0)
```

```
     STORAGE FOR VARIABLES APPEARING IN COMMON SENTENCES

             DEC   OCT               DEC   OCT               DEC   OCT               DEC   OCT
         H 32546 77442          HP 32561 77461         GAM 32559 77457        GAMQ 32558 77456
     GAMMA 32534 77426          EC 32560 77460           C 32562 77462          OI 32562 77462
        OM 32557 77455         OMU 32532 77424        PHIO 32553 77451       PHMAX 32535 77427
        PP 32548 77444          QP 32547 77443          RE 32533 77425           R 32545 77441
      SIGP 32550 77446         SIG 32544 77440         TBL 32531 77423         THO 32552 77450
      THPO 32551 77447        VRVV 32549 77445

NAMES OF ARITHMETIC STATEMENT FUNCTIONS WITH CORRESPONDING INTERNAL FORMULA NUMBERS AND OCTAL LOCATIONS

             IFN   LOC               IFN   LOC               IFN   LOC               IFN   LOC
      PRED     2 01517        SIMP     3 01533       RELER     4 01543

        EXTERNAL FORMULA NUMBERS WITH CORRESPONDING INTERNAL FORMULA NUMBERS AND OCTAL LOCATIONS

      EFN    IFN   LOC        EFN    IFN   LOC        EFN    IFN   LOC        EFN    IFN   LOC
        1     14 00000         23     18 00053         21     20 00063         22     22 00067
        3     50 00236          3     51 00236         31     53 00244         32     63 00313
        5     65 00330         51     69 00354         52     72 00377         53     74 00432
       54     77 00446          6     78 00452         61     79 00457         62     84 00474
        2     91 00000         14     92 00525         14     94 00534         17     95 00000
       15     97 00537         63    101 00557         64    106 00610         65    111 00644
       66    114 00714         67    115 00716         68    130 00753         69    152 00000
        9    153 01052          7    155 01056         71    161 01117         72    166 01165
       73    170 01205         10    172 01230         11    173 01235         16    175 01261
        4    178 01267         12    180 01272         13    181 01277

                                          STORAGE NOT USED BY PROGRAM

             DEC   OCT               DEC   OCT
            3854 07416             32362 77152

                                     LOCATIONS OF NAMES IN TRANSFER VECTOR

             DEC   OCT               DEC   OCT               DEC   OCT               DEC   OCT
     DIFFC     1 00001         COS     2 00002       ATMOS     3 00003        SQRT     4 00004
     (FIL)     5 00005      (IOH)O     7 00007       (LEV)     8 00010       (RTN)     0 00000
     (STH)     6 00006
```

```
          STORAGE LOCATIONS FOR VARIABLES APPEARING IN DIMENSION AND EQUIVALENCE SENTENCES

          DEC   OCT                DEC   OCT                DEC   OCT                DEC   OCT
     CO    962 01702        N    3846 07406       PHI    965 01705      PRBL   3853 07415
   SAV1   3845 07405      SAV    3845 07405      VMAX    959 01677         V    953 01671

      STORAGE LOCATIONS FOR VARIABLES NOT APPEARING IN DIMENSION,EQUIVALENCE OR COMMON SENTENCES

          DEC   OCT                DEC   OCT                DEC   OCT                DEC   OCT
  FNRAT    917 01625      ERR     916 01624     DPTST    915 01623      DPPR    914 01622
  DPMIN    913 01621    DPLIM     912 01620       DPC    911 01617         M    910 01616
   NRAT    909 01615      NS8     908 01614        NS    907 01613        NT    906 01612
   PHDP    905 01611    PHEND     904 01610      PHPR    903 01607        RP    902 01606
   TEST    901 01605

                  STORAGE LOCATIONS FOR SYMBOLS NOT APPEARING IN SOURCE PROGRAM

          DEC   OCT                DEC   OCT                DEC   OCT                DEC   OCT
 E)M       281 00431    C)1G4     906 01604    A)1G4     838 01506    8)25      776 01410
 8)H       780 01414    8)2       786 01422    8)1       837 01505    1)        896 01600
 1)1       893 01575    2)        705 01301    3)        710 01306    4)1       889 01571
 6)        727 01327    7)        888 01570

                           SUBROUTINES NOT PUNCHED FROM LIBRARY
 (LEV)       (IOH)O      (STH)      (FIL)        SQRT       ATMOS        COS        DIFFC
 (RTN)
```

```
      SUBROUTINE DIFFC(PHI,V)                                         1M091174
      DIMENSION C(200),V(3)                                           1M091176
      COMMON C                                                        1M091178
      EQUIVALENCE (C(32),TBL(1),SIG(14),R(15),H(16),QP(17),           1M091180
     2 PP(18),VRVV(19),SIGP(20),THPO(21),THO(22),PHIO(23),            1M091182
     3 EC(30),HP(31),OI(32),PHMAX(5),OM(27),GAMQ(28),GAM(29))         1M091184
     4 ,(GAMMA(4),RE(3),OMU(2),C(32) )                                1M091186
      SINP=SINF(PHI)                                                  1M091188
      COSP=COSF(PHI)                                                  1M091190
      GAMMA=EC*SINP/(1.+EC*COSP)                                      1M091192
      EOMCP=EC*(1.-COSP)                                              1M091194
      RV=(HP+RE)(1.+EC)/(1.+EC*COSP)                                  1M091196
      H=RV-RE                                                         1M091198
      CALL ATMOS(H,TBL,R)                                             1M091200
      RRVV=SIG*VRVV/SIGP                                              1M091202
      TWOQ=QP*RRVV*(1.+EOMCP)-EC*SINP                                 1M091204
      V(1)=TWOQ+TWOQ                                                  1M091206
      PTIL=PP*RRVV*VRVV*(1.+EOMCP+EOMCP)                              1M091208
      V(2)=OM*(1.-EC*COSP)/.3333 3333+PTIL                            1M091210
      V(3)=GAMMA*PTIL+TWOQ+TWOQ                                       1M091212
      RETURN                                                          1M091214
      END(0,0,0,0,0)
```

```
    STORAGE FOR VARIABLES APPEARING IN COMMON SENTENCES

            DEC   OCT                DEC   OCT                DEC   OCT                 DEC   OCT
        H 32546 77442           HP 32561 77461          GAM 32559 77457          GAMO 32558 77456
    GAMMA 32534 77426           EC 32560 77460            C 32562 77462            OI 32562 77462
       OM 32557 77455          OMU 32532 77424         PHIO 32553 77451         PHMAX 32535 77427
       PP 32548 77444           QP 32547 77443           RE 32533 77425             R 32545 77441
     SIGP 32550 77446          SIG 32544 77440          TBL 32531 77423           THO 32552 77450
     THPO 32551 77447         VRVV 32549 77445

                                          STORAGE NOT USED BY PROGRAM

            DEC   OCT                DEC   OCT
            133 00205              32362 77152

                                 LOCATIONS OF NAMES IN TRANSFER VECTOR

            DEC   OCT                DEC   OCT                DEC   OCT                DEC   OCT
    COS       1 00001    ATMOS         0 00000    SIN         2 00002

     STORAGE LOCATIONS FOR VARIABLES NOT APPEARING IN DIMENSION,EQUIVALENCE OR COMMON SENTENCES

            DEC   OCT                DEC   OCT                DEC   OCT                DEC   OCT
    EOMCP   132 00204     COSP       131 00203    PTIL      130 00202    RRVV      129 00201
       RV   128 00200     SINP       127 00177    TWOQ      126 00176

                     STORAGE LOCATIONS FOR SYMBOLS NOT APPEARING IN SOURCE PROGRAM

            DEC   OCT                DEC   OCT                DEC   OCT                DEC   OCT
    9)      121 00171      1)        123 00173     3)       114 00162     6)       116 00164
    7)      122 00172

                                        SUBROUTINES NOT PUNCHED FROM LIBRARY
      SIN         COS          ATMOS
```

```
C     CHAIN(1) PROGRAM TO SCALE FOR PUNCHING                              1M091800
      DIMENSION C(20)                                                     1M091801
      COMMON C                                                            1M091802
      IF DIVIDE CHECK 1,2                                                 1M091803
 2    C(20)=0.                                                            1M091804
 1    CALL FXRCH(C)                                                       1M091805
      CALL SCALE                                                          1M091806
      GO TO 1                                                             1M091807
      END(0,0,0,0,0)
 STORAGE FOR VARIABLES APPEARING IN COMMON SENTENCES

        DEC   OCT               DEC   OCT               DEC   OCT               DEC   OCT
     C 32562 77462

     EXTERNAL FORMULA NUMBERS WITH CORRESPONDING INTERNAL FORMULA NUMBERS AND OCTAL LOCATIONS

  EFN   IFN   LOC         EFN   IFN   LOC         EFN   IFN   LOC         EFN   IFN   LOC
     2     4 00004           1     5 00006           1     6 00006
                                    STORAGE NOT USED BY PROGRAM

        DEC   OCT               DEC   OCT
          20 00024             32542 77436
                              LOCATIONS OF NAMES IN TRANSFER VECTOR

        DEC   OCT               DEC   OCT               DEC   OCT               DEC   OCT
 FXRCH     1 00001       SCALE     0 00000
                  STORAGE LOCATIONS FOR SYMBOLS NOT APPEARING IN SOURCE PROGRAM

        DEC   OCT               DEC   OCT               DEC   OCT               DEC   OCT
3)        14 00016      6)        15 00017
                                    SUBROUTINES NOT PUNCHED FROM LIBRARY
 FXRCH       SCALE
```

```
        SUBROUTINE SCALE                                                   1M091820
        DIMENSION C(20),SAV(8,120),SAV1(960),PNCH(121,16),IPNCH(121,16),   1M091822
      2 SPNCH(5,16),PRBL(7),ISPNCH(5,16)                                   1M091824
        COMMON C                                                           1M091826
        EQUIVALENCE (SAV,SAV1),(PNCH,IPNCH),(SPNCH,ISPNCH)                 1M091827
        WRITE OUTPUT TAPE 6,1,(C(I),I=1,18)                                1M091828
  1     FORMAT (1H16X11HTAPE RECORD5X26HWEIGHT PAR    WEIGHT GAMMA         1M091830
      2 4X9HWEIGHT H16X9HWEIGHT H2/F18.0,1PE16.6,3E15.6/                   1M091832
      3 33HOSCALING - MIN VALUE   MIN COUNTS5X9HMAX VALUE6X                1M091834
      4 39HMAX COUNTS    EDGE COUNTS    CARD FIELD/                        1M091836
      5 9H FUNCTIONOPF11.3,F13.0,F14.3,F16.0,F15.0,F14.0/                  1M091838
      6 9H ARGUMENT F11.3,F13.0,F14.3,F16.0,F15.0,F14.0/                   1M091840
      7 10HODECK ID -F7.0)                                                 1M091842
        J=C(11)+.5                                                         1M091844
  3     IF (INTF(C(20)+.5)-INTF(C(1)+.5) ) 2,60,2                          1M091846
  2     READ TAPE 2,C(20),NS8,(SAV1(I),I=1,NS8),(PRBL(I),I=1,7)            1M091848
        IF (C(20)) 4,3,3                                                   1M091850
  4     REWIND 2                                                           1M091852
        IF (SENSELIGHT 1) 5,6                                              1M091854
  5     SPNCH(1,J)=SPNCH(3,J)                                              1M091856
        GO TO 69                                                           1M091858
  6     SENSELIGHT 1                                                       1M091860
        GO TO 2                                                            1M091862
  60    IF (SENSELIGHT 1) 61,61                                            1M091864
  61    WH1=C(4)                                                           1M091866
        WH2=C(5)                                                           1M091868
        WP=ABSF(C(2) )                                                     1M091870
        IF (C(2) ) 62,63,63                                                1M091872
  62    WH1=PRBL(2)                                                        1M091874
        WH2=PRBL(3)                                                        1M091876
  63    NS=NS8/8                                                           1M091878
        CALL WRITE(6HOD WHS,PRBL,3)                                        1M091879
        DO 64 I=1,5                                                        1M091880
  64    SPNCH(I,J)=C(I+5)                                                  1M091882
        PNCH(1,J)=WH1                                                      1M091884
        DO 65 I=1,NS                                                       1M091886
  65    PNCH(I+1,J)=(C(3)*SAV(2,I)+WP*SAV(7,I)+WH1*SAV(3,I)                1M091888
      2 +WH2*SAV(5,I))*57.295780                                           1M091890
        N=NS+1                                                             1M091892
        CALL WRITE(6HOFNCTN,PNCH(1,J),N)                                   1M091894
        IF (C(17) ) 69,69,66                                               1M091896
  66    J=C(17)+.5                                                         1M091898
```

```
      DO 67 I=1,5                                                       1M091900
   67 SPNCH(I,J)=C(I+11)                                                1M091902
      PNCH(1,J)=0.                                                      1M091904
      DO 68 I=1,NS                                                      1M091906
   68 PNCH(I+1,J)=57.295780*SAV(1,I)                                    1M091908
      CALL WRITE (6H0  ARG,PNCH(1,J),N)                                 1M091910
   69 IF (C(18) ) 9,9,7                                                 1M091912
    7 ID=C(18)+.5                                                       1M091914
      DO 72 J=1,16                                                      1M091916
      ISPNCH(5,J)=SPNCH(5,J)                                            1M091918
      DO 71  I=1,N                                                      1M091920
   71 IPNCH(I,J)=XMINOF(ISPNCH(5,J),NPSCL(PNCH(I,J),SPNCH(1,J) ) )      1M091922
      SPNCH(3,J)=SPNCH(1,J)                                             1M091923
   72 ISPNCH(4,J)=0                                                     1M091924
      N1=N+1                                                            1M091926
      N2=N1+1                                                           1M091928
      PUNCH 73,(ID,I,(IPNCH(I,J),J=1,16),I=1,N),ID,N1,                  1M091930
     2 (ISPNCH(4,J),J=1,16),ID,N2,(ISPNCH(5,J),J=1,16)                  1M091932
   73 FORMAT ( 1H0I3,17I4)                                              1M091934
      PUNCH 74,ID                                                       1M091936
   74 FORMAT ( 1H-I3/1H )                                               1M091938
      C(18)=0.                                                          1M091940
    9 RETURN                                                            1M091942
      END(0,0,0,0,0)
```

```
STORAGE FOR VARIABLES APPEARING IN COMMON SENTENCES

           DEC   OCT                DEC   OCT                DEC   OCT                DEC   OCT
         C 32562 77462

     EXTERNAL FORMULA NUMBERS WITH CORRESPONDING INTERNAL FORMULA NUMBERS AND OCTAL LOCATIONS

    EFN   IFN   LOC          EFN   IFN   LOC          EFN   IFN   LOC          EFN   IFN   LOC
      1    11 00000            3    13 00053            2    14 00066            2    23 00111
      4    25 00116            4    25 00116            5    27 00122            6    29 00125
     60    31 00127           61    32 00131           62    36 00143           63    38 00147
     64    42 00175           65    45 00222           66    50 00270           67    52 00316
     68    55 00343           69    58 00363            7    59 00367           71    63 00421
     71    64 00432           72    66 00455           73    85 00000           74    89 00000
      9    91 00574

                               STORAGE NOT USED BY PROGRAM

           DEC   OCT                DEC   OCT
          3498 06652              32542 77436

                               LOCATIONS OF NAMES IN TRANSFER VECTOR

           DEC   OCT                DEC   OCT                DEC   OCT                DEC   OCT
    NPSCL    1 00001     WRITE        2 00002     (FIL)        4 00004     (IOH)O       6 00006
    (LEV)    7 00007     (RTN)        3 00003     (SCH)        0 00000     (STH)        5 00005

      STORAGE LOCATIONS FOR VARIABLES APPEARING IN DIMENSION AND EQUIVALENCE SENTENCES

           DEC   OCT                DEC   OCT                DEC   OCT                DEC   OCT
   ISPNCH  601 01131     IPNCH     2537 04751      PNCH     2537 04751      PRBL      521 01011
    SAV1  3497 06651       SAV     3497 06651     SPNCH      601 01131

    STORAGE LOCATIONS FOR VARIABLES NOT APPEARING IN DIMENSION,EQUIVALENCE OR COMMON SENTENCES

           DEC   OCT                DEC   OCT                DEC   OCT                DEC   OCT
        I  514 01002        ID      513 01001         J      512 01000        N1      511 00777
       N2  510 00776         N      509 00775       NS8      508 00774        NS      507 00773
      WH1  506 00772       WH2      505 00771        WP      504 00770

                 STORAGE LOCATIONS FOR SYMBOLS NOT APPEARING IN SOURCE PROGRAM
```

```
              DEC   OCT                    DEC   OCT                    DEC   OCT                    DEC   OCT
E)17          379 00573       D)10A         81 00121       C)102        503 00767       C)100        502 00766
B)            491 00753       A)102        482 00742       A)100        473 00731       8)2A         400 00620
8)29          403 00623       8)1          472 00730       1)           497 00761       2)           384 00600
3)            390 00606       6)           393 00611

                                  SUBROUTINES NOT PUNCHED FROM LIBRARY
  (LEV)       (IOH)O       (STH)       (FIL)       (RTN)       WRITE       NPSCL       (SCH)
```

APPENDIX B

PROGRAM DESCRIPTION - ATMOSPHERE SUBROUTINE
NORTH AMERICAN AVIATION, INC. COLUMBUS
ENGINEERING PROGRAM DESCRIPTION

1. Identification

a. Atmosphere Subroutine, 1F113
b. Sarah Crooke, August 1959
c. Aero Advanced Design Group - 214
d. Fortran Source Deck is up to date.

2. Purpose

Calculates standard atmosphere properties at a specified altitude.

3. Restrictions

Fortran 2 subroutine subprogram.

4. Method

Table look-up of properties at base geopotentials followed by application of the ideal gas law.

5. Use

Subroutine is reached by Fortran Statement

```
CALL ATMØS (H, TBL, R)
```

where H is the input altitude
TBL is the table of base geopotentials and properties
R is the results array.

6. Coding Information

a. This routine occupies 309 Locations.
b. Inferior Subroutines are Library routines DUMP, EXP, SQRT, and the Fortran system routine EXP(3.

7. Data Arrangement

a. The array TBL must contain the following data:

Location	Description	Symbol
1	Sea Level Temperature, °R	T_0
2	Sea Level Pressure, lb/ft^2	P_0

Location	Description	Symbol
3	Sea Level Gas Constant, $ft^2/sec^{2}\,^{o}R$	R_o
4	Sea Level Density, $slugs/ft^3$	ρ_o
5	Number of Base Geopotentials in Table	n
6 ⋮ $5 + n$	Base Geopotentials, ft	h_b
$6 + n$ ⋮ $5 + 2n$	Base Gas Constants, $ft^2/sec^{2}\,^{o}R$	R_b
$6 + 2n$ ⋮ $5 + 3n$	Base Temperatures, ^{o}R	T_b
$6 + 3n$ ⋮ $5 + 4n$	Base Specific Heat Ratios, dimensionless	γ_b
$6 + 4n$ A_1 $7 + 4n$ B_1 ⋮ ⋮ $4 + 6n$ A_n $5 + 6n$ B_n	$A_i \times 10^{b_i}$ is the ith Base Pressure Ratio, dimensionless	δ_b

b. On return the array R will contain the following data:

Location	Description	Symbol
1	Geopotential Altitude, ft	h
2	Density Ratio, dimensionless	σ
3	Speed of Sound, ft/sec	a
4	Speed of Sound, knots	A
5	Pressure Ratio, dimensionless	δ
6	Local Acceleration of Gravity, ft/sec^2	g

Location	Description	Symbol
7	Temperature, °R	T
8	Gas Constant, ft^2/sec^2°R	R
9	Incompressible Dynamic Pressure at Mach 1, lb/ft^2	G
10	Ratio of Specific Heats, dimensionless	γ

8. Formulas

In the formulas below H is the input altitude. The remaining symbols are those listed in 7 above. The subscript B refers to the value of the quantity at the ith base geopotential altitude, where

$$i = 1 \quad \text{if} \quad h < h_2 \,,$$

$$i = j \quad \text{if} \quad h_j \leq h < h_{j+1} \,, \qquad j = 2, \dots, n-1 \,,$$

$$i = n-1 \ \text{if} \quad h_n \leq h .$$

The prefix Δ denotes the increment of the quantity from the ith to the $(i+1)$th base geopotential altitude.

$$h = \frac{20\,891\,000\,H}{H + 20\,891\,000}$$

$$T = T_B + (h - h_B)\,\frac{\Delta T}{\Delta h}$$

$$R = R_B + (h - h_B)\,\frac{\Delta R}{\Delta h}$$

$$\delta = \delta_B \left[\frac{R_B}{R} \quad \frac{T}{T_B}\right]^{32.174\,049\,\Delta h / (T_B \Delta R - R_B \Delta T)} ,$$

$$\text{if } T_B \Delta R - R_B \Delta T \neq 0$$

$$= \delta_B \, e^{-32.174\,049\,(h - h_B)/\Delta h} ,$$

$$\text{if } T_B \Delta R - R_B \Delta T = 0$$

$$\sigma = \delta \frac{T_0}{T} \frac{R_0}{R}$$

$$\gamma = \gamma_B + (h - h_B) \frac{\Delta \gamma}{\Delta h}$$

$$g = 32.174049 \left[\frac{20\,891\,000}{H + 20\,891\,000} \right]^2$$

$$a = \sqrt{\frac{\gamma P_0 \delta}{\rho_0 \sigma}}$$

$$A = \frac{a}{1.6878099}$$

$$G = \frac{1}{2} (\rho_0 \sigma) a^2$$

```
      SUBROUTINE ATMOS(H,TABLE,RES)                                     1F040100
      DIMENSION TABLE(100),RES(10),R(10)                                1F040105
      R(1)=H*20891000./(H+20891000.)                                    1F040110
      I=TABLE(5)                                                        1F040115
      IF(I-1)5,5,10                                                     1F040120
 5    R(1)=DUMPF(1)                                                     1F040125
 10   DO20K=2,I                                                         1F040130
      IF(TABLE(K+5)-R(1))20,15,15                                       1F040135
 20   CONTINUE                                                          1F040140
      K=I                                                               1F040145
 15   HB=TABLE(K+4)                                                     1F040150
      R(3)=TABLE(K+5)-HB                                                1F040155
      J=I+K                                                             1F040160
      RB=TABLE(J+4)                                                     1F040165
      DRDH=(TABLE(J+5)-RB)/R(3)                                         1F040170
      J=J+I                                                             1F040175
      TB=TABLE(J+4)                                                     1F040180
      DTDH=(TABLE(J+5)-TB)/R(3)                                         1F040185
      J=J+I                                                             1F040190
      GAMB=TABLE(J+4)                                                   1F040195
      DGDH=(TABLE(J+5)-GAMB)/R(3)                                       1F040200
      J=J+I+K                                                           1F040205
      PBPO=TABLE(J+2)*10.**TABLE(J+3)                                   1F040210
      R(9)=R(1)-HB                                                      1F040215
      R(7)=TB+DTDH*R(9)                                                 1F040220
      R(8)=RB+DRDH*R(9)                                                 1F040225
      R(10)=TB*DRDH-RB*DTDH                                             1F040230
      IF(R(10))30,25,30                                                 1F040235
 25   R(2)=EXPF(-32.174049*R(9)/(R(7)*R(8)))                            1F040240
      GOTO35                                                            1F040245
 30   R(2)=(RB*R(7)/(R(8)*TB))**(32.174049/R(10))                       1F040250
 35   R(5)=R(2)*PBPO                                                    1F040255
      R(2)=R(5)*TABLE(1)*TABLE(3)/(R(7)*R(8))                           1F040260
      R(10)=GAMB+DGDH*R(9)                                              1F040265
      R(6)=32.174049*(20891000./(20891000.+H))**2                       1F040270
      R(3)=SQRTF(R(10)*R(5)*TABLE(2)/(TABLE(4)*R(2)))                   1F040275
      R(4)=R(3)/1.6878099                                               1F040280
      R(9)=.5*R(3)*R(3)*TABLE(4)*R(2)                                   1F040285
      DO40I=1,10                                                        1F040290
 40   RES(I)=R(I)                                                       1F040295
      RETURN                                                            1F040300
      END(0,0,0,0,0)
```

```
          EXTERNAL FORMULA NUMBERS WITH CORRESPONDING INTERNAL FORMULA NUMBERS AND OCTAL LOCATIONS

   EFN   IFN   LOC            EFN   IFN   LOC            EFN   IFN   LOC            EFN   IFN   LOC
     5     6 00106             10     7 00113             20     9 00132             15    11 00141
    25    29 00254             30    31 00272             35    32 00312             40    40 00401

                                          STORAGE NOT USED BY PROGRAM

         DEC   OCT                 DEC   OCT
         309 00465               32562 77462

                                       LOCATIONS OF NAMES IN TRANSFER VECTOR

           DEC   OCT                 DEC   OCT                 DEC   OCT                 DEC   OCT
  EXP(3      2 00002         EXP       1 00001         DUMP      3 00003         SQRT      0 00000

        STORAGE LOCATIONS FOR VARIABLES APPEARING IN DIMENSION AND EQUIVALENCE SENTENCES

         DEC   OCT                 DEC   OCT                 DEC   OCT                 DEC   OCT
     R   308 00464

     STORAGE LOCATIONS FOR VARIABLES NOT APPEARING IN DIMENSION,EQUIVALENCE OR COMMON SENTENCES

         DEC   OCT                 DEC   OCT                 DEC   OCT                 DEC   OCT
     I   298 00452           HB    297 00451         GAMB    296 00450         DTDH    295 00447
  DRDH   294 00446         DGDH    293 00445            J    292 00444            K    291 00443
  PBPO   290 00442           RB    289 00441           TB    288 00440

                        STORAGE LOCATIONS FOR SYMBOLS NOT APPEARING IN SOURCE PROGRAM

         DEC   OCT                 DEC   OCT                 DEC   OCT                 DEC   OCT
 E)9     170 00252      E)5         86 00126      C)G2     287 00437      C)G1     286 00436
 9)      277 00425      1)         283 00433      2)       265 00411      3)       267 00413
 6)      272 00420      7)         282 00432

                                       SUBROUTINES NOT PUNCHED FROM LIBRARY
  DUMP         EXP(3          EXP          SQRT
```

APPENDIX C

PROGRAM DESCRIPTION - FIXED DECIMAL CARD READ ROUTINE WITH ERROR DETECTION NORTH AMERICAN AVIATION, INC. COLUMBUS ENGINEERING PROGRAM DESCRIPTION

1. Identification

   a. Fixed Decimal Card Read Routine with Error Detection - Deck NCF-FXRCH
   b. O. C. Juelich, March 1960
   c. Aero Advanced Design Group - 214
   d. Fortran Source Deck is up to date.

2. Purpose

   a. Load Input Data for Fortran II programs.
   b. Permit resumption of loading after cards with zero or fractional origin are found.
   c. List such error cards in the print-out.

3. Restrictions

   a. Reads input tape 5, writes output tape 6 in case of error.
   b. Data cards are written on Form 114-C-17 (Fortran Fixed 10 Digit Decimal Data).
   c. Uses Fortran II Error Procedure.

4. Method

   a. Card images are read from the input tape. The first word specifies the location to which the data is to be transmitted, and whether the data set is complete.

   b. Filled-in card fields are transmitted to the calling program, blank fields are not transmitted.

   c. Error Card images are written on the output tape. If error cards are found the error procedure is invoked at the end of the data set.

5. Use

   a. Subroutine is reached by the Fortran Statement:

   CALL FXRCH (D) .

   b. A card image is read. The first word on the card is the subscript n, written with decimal point. The $(i+2)$th word on the card; i = 0, 1, 2, 3, 4; is transmitted to location $D(|n|+i)$ as a floating point number unless it is blank or -0. If the $(i+2)$th word is blank or -0 location $D(|n|+i)$ is unmodified. If n is positive this procedure is repeated. If n is negative control returns to the calling program. (Subscripting as in EQUIVALENCE statements.)

6. Coding Information

a. This routine requires 174 locations.

b. Inferior subroutines are the Library routine ERRØR and Fortran System Subroutines.

```
C     FIXED DECIMAL CARD READ ROUTINE WITH ERROR DETECTION              1F126000
      SUBROUTINE FXRCH(D)                                               1F126010
      DIMENSION A(2),C(6),D(6)                                          1F126020
      I=1                                                               1F126030
    1 READ INPUT TAPE 5,101,(C(J),J=1,6),A(1),A(2)                      1F126040
      J=ABSF(C(1))                                                      1F126050
    4 IF (J) 2,99,2                                                     1F126060
    2 IF (C(1)-INTF(C(1))) 99,3,99                                      1F126070
    3 DO 13 K=2,6                                                       1F126080
      IF (C(K)) 12,11,12                                                1F126090
   11 IF (SIGNF(1.,C(K))) 13,13,12                                      1F126100
   12 D(J)=C(K)                                                         1F126110
   13 J=J+1                                                             1F126120
   14 IF (SIGNF(1., C(1))) 20,20,1                                      1F126130
   20 GO TO (40,30),I                                                   1F126140
   30 CALL ERROR                                                        1F126150
   40 RETURN                                                            1F126160
   99 GO TO (97,98),I                                                   1F126170
   97 I=2                                                               1F126180
      WRITE OUTPUT TAPE 6,102                                           1F126190
   98 WRITE OUTPUT TAPE 6,103,(C(J),J=1,6),A(1),A(2)                    1F126200
      GO TO 14                                                          1F126210
  101 FORMAT (6F12.0,A6,A2)                                             1F126220
  102 FORMAT (10H1BAD CARDS)                                            1F126230
  103 FORMAT (F16.8,5E16.8,A6,A2)                                       1F126240
      FREQUENCY 4(0,0,1),2(0,1,0),11(1,0,1),14(1,0,5),20(1,0) ,         1F126245
     2 99(1,0)                                                          1F126248
      END(0,0,0,0,0)
```

```
       EXTERNAL FORMULA NUMBERS WITH CORRESPONDING INTERNAL FORMULA NUMBERS AND OCTAL LOCATIONS

  EFN   IFN  LOC          EFN   IFN  LOC          EFN   IFN  LOC          EFN   IFN  LOC
    1     4 00027           1     9 00050           4    11 00063           2    12 00065
    3    13 00075          11    15 00104          12    16 00112          13    17 00116
   14    18 00126          20    19 00134          30    20 00140          40    21 00143
   99    23 00151          97    24 00153          98    27 00172          98    32 00213
  101    34 00000         102    35 00000         103    36 00000

                                   STORAGE NOT USED BY PROGRAM

        DEC   OCT                DEC   OCT
        174 00256              32562 77462

                                 LOCATIONS OF NAMES IN TRANSFER VECTOR

          DEC   OCT                DEC   OCT                DEC   OCT                DEC   OCT
 ERROR      3 00003      (FIL)       0 00000      (IOH)I      6 00006      (IOH)O      2 00002
 (LEV)      7 00007      (RTN)       4 00004      (STH)       1 00001      (TSH)       5 00005

      STORAGE LOCATIONS FOR VARIABLES APPEARING IN DIMENSION AND EQUIVALENCE SENTENCES

          DEC   OCT                DEC   OCT                DEC   OCT                DEC   OCT
 C        171 00253      A         173 00255

    STORAGE LOCATIONS FOR VARIABLES NOT APPEARING IN DIMENSION,EQUIVALENCE OR COMMON SENTENCES

          DEC   OCT                DEC   OCT                DEC   OCT                DEC   OCT
 I        165 00245      J         164 00244

                        STORAGE LOCATIONS FOR SYMBOLS NOT APPEARING IN SOURCE PROGRAM

          DEC   OCT                DEC   OCT                DEC   OCT                DEC   OCT
 E)H      121 00171      E)F       103 00147      E)D        95 00137      E)7        66 00102
 D)409     77 00115      D)401      22 00026      D)20F     104 00150      D)20C      91 00133
 D)108     85 00125      C)G3      163 00243      C)G2      162 00242      C)G0      161 00241
 9)       160 00240      8)37      153 00231      8)36      156 00234      8)35      159 00237
 2)       142 00216      3)        144 00220      6)        145 00221

                                SUBROUTINES NOT PUNCHED FROM LIBRARY
 (LEV)        (IOH)I        (TSH)         (RTN)       ERROR       (IOH)O       (STH)        (FIL)
```

APPENDIX D

PROGRAM DESCRIPTION - PUNCH SCALING FUNCTION
NORTH AMERICAN AVIATION, INC. COLUMBUS
ENGINEERING PROGRAM DESCRIPTION

1. Identification

a. Punch Scaling Function, Deck - NCF NPSCL.
b. J. B. Burnett, May 1960.
c. Digital Computing Group - 331.
d. Source Deck is up to date.

2. Purpose

Converts floating point data to fixed point data which is scaled suitably for automatic plotting equipment.

3. Restrictions

Operates as a Fortran II Function Subprogram.

4. Method

Linear Scaling.

5. Use

a. Reached through a Fortran Statement such as

```
IPNCH = NPSCL (A, S)
```

where A is the number to be scaled

S is an array of dimension 4 containing the scale table.

S(2) holds the plotter counts corresponding to the left or bottom edge of the graph
S(4) holds the plotter counts corresponding to the right or top edge of the graph
S(1) holds the value of A corresponding to S(2)
S(3) holds the value of A corresponding to S(4)

b. S(2) and S(4) must be between 0.0 and 999.0,
S(3) must exceed S(1). If the array S does not meet these specifications IPNCH will be set to -0.

c. If A is less than S(1), IPNCH will be set to 0.
If A exceeds S(3) IPNCH will be set to 999.
(The user must make provision to confine the plotting head to the plotting bed.)

d. It is expected that the plotting machine be set to 20 counts per centimeter, with (0, 0) counts at the lower left hand corner of the grid.

6. Coding Information

a. This routine occupies 119 locations.
b. There are no inferior subroutines.

```
      FUNCTION NPSCL(A,S)                                               NPSCL300
      DIMENSION S(4)                                                    NPSCL305
      IF (S(3)-S(1)) 98,98,1                                            NPSCL310
    1 DEN=S(3)-S(1)                                                     NPSCL315
      IF (S(4)) 98,2,2                                                  NPSCL320
    2 IF (S(2)) 98,3,3                                                  NPSCL325
    3 IF (S(4)-999.) 14,14,98                                           NPSCL330
   14 IF (S(2)-999.) 4,4,98                                             NPSCL332
    4 ENUM=S(4)-S(2)                                                    NPSCL335
      IF (A-S(1)) 99,5,5                                                NPSCL340
    5 IF (A-S(3)) 6,6,100                                               NPSCL345
    6 NPSCL=(A-S(1) )*ENUM/DEN+S(2)+.5                                  NPSCL350
      GO TO 101                                                         NPSCL355
   98 NPSCL=-0                                                          NPSCL360
      GO TO 101                                                         NPSCL365
   99 NPSCL=0                                                           NPSCL370
      GO TO 101                                                         NPSCL375
  100 NPSCL=999                                                         NPSCL380
  101 RETURN                                                            NPSCL385
      END(0,0,0,0,0)
      EXTERNAL FORMULA NUMBERS WITH CORRESPONDING INTERNAL FORMULA NUMBERS AND OCTAL LOCATIONS
  EFN    IFN   LOC         EFN   IFN   LOC         EFN    IFN   LOC         EFN   IFN   LOC
    1      4 00055           2     6 00064           3      7 00070          14     8 00074
    4      9 00100           5    11 U0110           6     12 00114          98    14 00131
   99     16 00134         100    18 00137         101     19 00141

                                       STORAGE NOT USED BY PROGRAM
        DEC   OCT                  DEC   OCT
        119 00167                32562 77462

     STORAGE LOCATIONS FOR VARIABLES NOT APPEARING IN DIMENSION,EQUIVALENCE OR COMMON SENTENCES
        DEC   OCT                 DEC   OCT                  DEC   OCT                  DEC   OCT
 ENUM   118 00166          DEN    117 00165         NPSCL    116 00164

                       STORAGE LOCATIONS FUR SYMBOLS NOT APPEARING IN SOURCE PROGRAM
        DEC   OCT                 DEC   OCT                  DEC   OCT                  DEC   OCT
9)      111 00157          2)     102 U0146          3)      104 00150          6)      106 00152
```

APPENDIX E

PROGRAM DESCRIPTION - OUTPUT TAPE WRITE ROUTINE
NORTH AMERICAN AVIATION, INC. COLUMBUS
ENGINEERING PROGRAM DESCRIPTION

1. Identification

a. Output Tape Write Routine, Deck - NCF WRITE.
b. O. C. Juelich, June 1960.
c. Research Group - 330.
d. Fortran Source Deck is up to date.

2. Purpose

Writes up to ten floating point numbers per line retaining seven digit accuracy for most numbers.

3. Restrictions

a. Writes output tape 6.
b. Intended primarily for numbers between 10^{-3} and 10^7 in magnitude. Seven digit accuracy is retained for numbers between 1 and 10^7.
c. Source Deck includes 704 SAP instructions.

4. Method

A Format Statement is generated, using F conversion for numbers less than 10^7 and E conversion for numbers 10^7 or larger in magnitude.

5. Use

Reached through a Fortran Statement such as

```
CALL WRITE (6HiIDENT, A, N)
```

where i is the pre-print space control character
IDENT is the indicative word for the first line of print. (The second line, if any, will be identified by an 11, the third by 21, etc.)
A is the first word to be printed.
N is the number of words to be printed, $N > 0$.

6. Coding Information

a. This routine requires 171 locations.
b. All inferior subroutines are part of the Fortran System.

```
      SUBROUTINE WRITE(A,F,N)                                    WRITE000
SFMTA ALF    (A6,                                                WRITE010
SFMTE ALF    E10.2,                                              WRITE020
SFMTF ALF    F10.0,                                              WRITE030
SFMTI ALF    (I6,                                                WRITE040
SFMTX ALF     )                                                  WRITE050
      DIMENSION F(25),FMT(12)                                    WRITE060
      EQUIVALENCE (B,IB)                                         WRITE070
      FMT(1)=FMTA                                                WRITE080
      B=A                                                        WRITE090
      J=1                                                        WRITE100
    1 K=XMINOF(J+9,N)                                            WRITE110
      DO 2 I=J,K                                                 WRITE120
      IT=I+2-J                                                   WRITE130
      X=ABSF(F(I) )                                              WRITE140
      L=-1                                                       WRITE150
      DO 3 M=1,7                                                 WRITE160
      IF (X-9 999 999.) 4,4,5                                    WRITE170
    4 X=10.*X                                                    WRITE180
    3 L=L+1                                                      WRITE190
    5 IF (L) 6,7,7                                               WRITE200
    6 FMT(IT)=FMTE                                               WRITE210
      GO TO 8                                                    WRITE220
S7    CLA L                                                      WRITE230
S     ARS 12                                                     WRITE240
S     ORA FMTF                                                   WRITE250
S     SLW FMT(IT)                                                WRITE260
    8 FMT(IT+1)=FMTX                                             WRITE270
    2 CONTINUE                                                   WRITE280
      WRITE OUTPUT TAPE 6,FMT,B,(F(I),I=J,K)                     WRITE290
      IF (K-N) 9,10,9                                            WRITE300
    9 J=J+10                                                     WRITE310
      IB=J                                                       WRITE320
      FMT(1)=FMTI                                                WRITE330
      GO TO 1                                                    WRITE340
   10 RETURN                                                     WRITE350
      CARDS                                                      WRITE360
```

```
        EXTERNAL FORMULA NUMBERS WITH CORRESPONDING INTERNAL FORMULA NUMBERS AND OCTAL LOCATIONS
  EFN    IFN   LOC          EFN    IFN   LOC          EFN    IFN   LOC          EFN    IFN   LOC
     1    12 00035             4    19 00077             3    20 00102             5    21 00112
     6    22 00117             7    24 00122             8    28 00126             2    29 00130
     9    38 00165            10    42 00175

                                       STORAGE NOT USED BY PROGRAM
          DEC   OCT                  DEC   OCT
          171 00253                32562 77462

                                    LOCATIONS OF NAMES IN TRANSFER VECTOR
          DEC   OCT                  DEC   OCT                  DEC   OCT                  DEC   OCT
  (FIL)     0 00000      (IOH)O      2 00002      (LEV)         3 00003      (STH)         1 00001
        STORAGE LOCATIONS FOR VARIABLES APPEARING IN DIMENSION AND EQUIVALENCE SENTENCES
          DEC   OCT                  DEC   OCT                  DEC   OCT                  DEC   OCT
    IB    170 00252         FMT      169 00251         B       170 00252
     STORAGE LOCATIONS FOR VARIABLES NOT APPEARING IN DIMENSION,EQUIVALENCE OR COMMON SENTENCES
          DEC   OCT                  DEC   OCT                  DEC   OCT                  DEC   OCT
    IT    157 00235          I       156 00234          J      155 00233          K      154 00232
    L     153 00231          M       152 00230          X      151 00227
                          STORAGE LOCATIONS FOR SYMBOLS NOT APPEARING IN SOURCE PROGRAM
          DEC   OCT                  DEC   OCT                  DEC   OCT                  DEC   OCT
  E)B      91 00133      E)8          77 00115      E)6          71 00107      D)601        27 00033
  D)407    73 00111      D)401        28 00034      C)200       150 00226      C)G2        149 00225
  C)G1    148 00224      9)          145 00221      1)          146 00222      2)          129 00201
  3)      138 00212      6)          140 00214

  (LEV)      (IOH)O       (STH)         SUBROUTINES NOT PUNCHED FROM LIBRARY
                                          (FIL)
```

APPENDIX F

SUMMARY OF LIBRARY ROUTINES

SQRT Evaluates the square root of a non-negative floating-point number. Invokes ERRØR if argument is less than zero.

CØS Evaluates the cosine of an angle expressed in radians in floating-point form.

SIN Evaluates the sine of an angle expressed in radians in floating-point form.

EXP Evaluates the exponential function (e^x) of a floating-point number less than 88. Invokes ERRØR is the argument exceeds 88.

ERRØR a. Produces a "back-trace" through the program, listing the point and subroutine at which the error was detected, the point at which the subroutine was called, up to the main program.

b. Turns on the Divide Check indicator and tests the first instruction of the main program. If this is "IF DIVIDE CHECK" control goes to this statement, otherwise DUMP is invoked.

CHAIN Loads a program identified by Record and Tape Number from Tape to Core and gives it control.

DUMP Gives a full Memory Print-out and invokes EXIT.

EXIT Returns control to the Monitor to end the job. (This function is also performed by the input tape reading system routine if an "End of File" is encountered on the input tape.)

APPENDIX G

SAMPLE INPUT AND OUTPUT

FORTRAN FIXED 10 DIGIT DECIMAL DATA

DECK NO.________ PROGRAMMER________________ DATE________ PAGE_____of_____ JOB NO.________

	NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH	
1	32.		TBL (1)	
13	518.688		T_0	°R
25	2116.2169		P_0	lb/ft²
37	1716.4827		R_0	ft²/sec² °R
49	.0023769917	73 80	ρ_0	slug/ft³
61	13.	1	n	
1	37.		TBL (6)	
13	-500.01197		h_B 1	ft
25	36089.2388		h_B 2	ft
37	82020.9973		h_B 3	ft
49	154199.475	73 80	h_B 4	ft
61	173884.514	2	h_B 5	ft
1	42.		TBL (11)	
13	259186.351		h_B 6	ft
25	295275.590		h_B 7	ft
37	344488.188		h_B 8	ft
49	524934.382	73 80	h_B 9	ft
61	557742.781	3	h_B 10	ft
1	47.		TBL (16)	
13	656167.978		h_B 11	ft
25	1283000.		h_B 12	ft
37	2623289.063		h_B 13	ft
49	1716.4827	73 80	R_B 1	ft²/sec² °R
61	1716.4827	4	R_B 2	ft²/sec² °R

FORTRAN FIXED 10 DIGIT DECIMAL DATA

DECK NO._____ PROGRAMMER_____ DATE_____ PAGE___ of___ JOB NO._____

	NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH	
1	52.		TBL (21)	
13	1716.4827		R_B 3	ft^2/sec^2 °R
25	1716.4827		R_B 4	ft^2/sec^2 °R
37	1716.4827		R_B 5	ft^2/sec^2 °R
49	1716.4827	73 80	R_B 6	ft^2/sec^2 °R
61	1716.4827	5	R_B 7	ft^2/sec^2 °R
1	57.		TBL (26)	
13	1716.4827		R_B 8	ft^2/sec^2 °R
25	1716.4827		R_B 9	ft^2/sec^2 °R
37	1716.4827		R_B 10	ft^2/sec^2 °R
49	1716.4827	73 80	R_B 11	ft^2/sec^2 °R
61	1716.4827	6	R_B 12	ft^2/sec^2 °R
1	62.		TBL (31)	
13	1716.4827		R_B 13	ft^2/sec^2 °R
25	520.471		T_B 1	°R
37	389.988		T_B 2	°R
49	389.988	73 80	T_B 3	°R
61	508.788	7	T_B 4	°R
1	67.		TBL (36)	
13	508.788		T_B 5	°R
25	298.188		T_B 6	°R
37	298.188		T_B 7	°R
49	406.188	73	T_B 8	°R
61	2386.188	8	T_B 9	°R

FORTRAN FIXED 10 DIGIT DECIMAL DATA

DECK NO. ______ PROGRAMMER ______ DATE ______ PAGE ___ of ___ JOB NO. ______

	NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH	
1	72.		TBL (41)	
13	2566.188		T_B 10	°R
25	2836.188		T_B 11	°R
37	4039.856		T_B 12	°R
49	6613.5328	73 80	T_B 13	
61	1.4011	9	γ_B 1	dimensionless
1	77.		TBL (46)	
13	1.4011		γ_B 2	dimensionless
25	1.4011		γ_B 3	dimensionless
37	1.4011		γ_B 4	dimensionless
49	1.4011	73 80	γ_B 5	dimensionless
61	1.4011	10	γ_B 6	dimensionless
1	82.		TBL (51)	
13	1.4011		γ_B 7	dimensionless
25	1.4025		γ_B 8	dimensionless
37	1.4195		γ_B 9	dimensionless
49	1.4268	73 80	γ_B 10	dimensionless
61	1.4545	11	γ_B 11	dimensionless
1	87.		TBL (56)	
13	1.5920		γ_B 12	dimensionless
25	1.65834		γ_B 13	dimensionless
37	1.0182020		δ_B 1 A	dimensionless
49	0.	73 80	× 10^8	
61	.2233588587	12	δ_B 2 A	dimensionless

FORTRAN FIXED 10 DIGIT DECIMAL DATA

DECK NO.______ PROGRAMMER______ DATE______ PAGE___ of___ JOB NO.______

		NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH	
1		92.		TBL (61)	
13		0.		$\times 10^8$	
25		.245605231		δ_B 3 A	dimensionless
37	–	1.		$\times 10^8$	
49		.11886576	73 80	δ_B 4 A	dimensionless
61	–	2.	13	$\times 10^8$	
1		97.		TBL (66)	
13		.57557334		δ_B 5 A	dimensionless
25	–	3.		$\times 10^8$	
37		.99623335		δ_B 6 A	dimensionless
49	–	5.	73 80	$\times 10^8$	
61		.10306942	14	δ_B 7 A	dimensionless
1		102.		TBL (71)	
13	–	5.		$\times 10^8$	
25		.73550492		δ_B 8 A	dimensionless
37	–	7.		$\times 10^8$	
49		.35726255	73 80	δ_B 9 A	dimensionless
61	–	8.	15	$\times 10^8$	
1		107.		TBL (76)	
13		.27866476		δ_B 10 A	dimensionless
25	–	8.		$\times 10^8$	
37		.14067391		δ_B 11 A	dimensionless
49	–	8.	73 80	$\times 10^8$	
61		.44521087	16	δ_B 12 A	dimensionless

FORTRAN FIXED 10 DIGIT DECIMAL DATA

DECK NO.______ PROGRAMMER______ DATE______ PAGE___ of___ JOB NO.______

		NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH	
1		112.		TBL (81)	
13	-	10.		$\times 10^B$	
25		.36222		δ_B 13 A	dimensionless
37	-	12.		$\times 10^B$	
49					
61			73 80		
			17		
1		28.			
13		22230100.		RECID	
25					
37		20902131.		r_E	ft
49		1.407698+16	73 80	μ = $1.407\,698 \times 10^{16}$	ft^3/sec^2
61					
			18		
1		7.			
13		.05		$d\nu_{min}$	degrees
25					
37		5.		$d\nu_{print}$	degrees
49					
61			73 80		
			19		
1		201.			
13		1.		n_i	
25		0.		i	degrees
37					
49					
61			73 80		
			20		

FORTRAN FIXED 10 DIGIT DECIMAL DATA

DECK NO.______ PROGRAMMER______ DATE______ PAGE___ of___ JOB NO.______

	NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	2 1 1 .		
13	1 .		n_h
25	1 . 6 + 0 6		$h = 1.6 \times 10^6$ ft.
37			
49		73 80	
61		2 1	
1	2 2 1 .		
13	2 .		n_ϵ
25	0 .		ϵ_1 dimensionless
37	. 0 2 5		ϵ_2
49		73 80	
61		2 2	
1	2 3 1 .		
13	1 .		n_Γ
25	1 .		Γ
37			
49		73 80	
61		2 3	
1	2 4 1 .		
13	1 .		n_{Γ_s}
25	0 .		Γ_s
37			
49		73 80	
61		2 4	

FORM 111-C-17

FORTRAN FIXED 10 DIGIT DECIMAL DATA

DECK NO.______ PROGRAMMER______________ DATE________ PAGE____of______ JOB NO.____________

		NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	–	2 5 1 .		Last Card of First Data Set.
13		1 .		h_M
25		0 .		M
37				
49			73 80	
61			2 5	
1	–	2 8 .		Last Card of Second Data Set.
13		0 .		RECID = 0, Mark "end" on Tape 2
25				
37				
49			73 80	
61			2 6	
1	–	2 8 .		Last Card of Third Data Set.
13	–	1 .		RECID < 0, exit to Card-Punch Program via "CHAIN."
25				
37				
49			73 80	
61			2 7	
1		1 .		
13		2 2 2 3 0 1 0 1 .		RECID
25		1 .		w_p
37		0 .		w_x
49		0 .	73 80	w_1
61		0 .	2 8	w_2

FORTRAN FIXED 10 DIGIT DECIMAL DATA

DECK NO.______ PROG. MMER______ DATE______ PAGE___ of___ JOB NO.______

		NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1		6.		
13	-	30.		f_{min}
25		200.		c_{min}
37		30.		f_{max}
49		800.		c_{max}
61		850.	29	c_{edge}
1		11.		
13		2.		F_f
25		0.		v_{min}
37		100.		d_{min}
49		360.		v_{max}
61		820.	30	d_{max}
1	-	16.		Last Card of Data Set.
13		850.		d_{edge}
25		1.		F_v
37		0.		CID (not ready to punch cards)
49				
61			31	
1		2.		
13		0.		w_p
25		1.		w_x
37				
49				
61			32	

FORTRAN FIXED 10 DIGIT DECIMAL DATA

DECK NO.________ PROGRAMMER ____________________ DATE ________ PAGE ____ of ____ JOB NO. ________

	NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	11.		
13	3.		F_f
25			
37			
49			
61		73 80	
		33	
1	-17.		Last Card of Data Set
13	0.		F_v (do not punch v - same argument field as before)
25			
37			
49			
61		73 80	
		34	
1			
13			
25			
37			
49			
61		73 80	
1			
13			
25			
37			
49			
61		73 80	

```
 PARAMETER LIST -
  OI =       0.
  HP =  1600000.
  EC =  0.          0.025000
 GAM =      1.000
GAMQ =      0.
  OM =  0.
```

```
    OI           HP            EC            GAM           GAMQ          OM
    0.           1600000.      0.            1.0000E 00    0.            0.

D PHI MIN    D PHI PRINT                                                RECORD ID
5.0000E-02   5.0000E 00                                                 22230100.

    PHI          H          RHO              GAMMA      ALPHA H1     ALPHA H2     ALPHA PA      N
     5.000      1600000.   5.0380E-15        0.           0.996        4.993        -0.          8
    10.000      1600000.   5.0390E-15        0.           0.983        9.944         0.          1
    15.000      1600000.   5.0380E-15        0.           0.962       14.810        -0.          1
    20.000      1600000.   5.0380E-15        0.           0.933       19.551         0.          1
    25.000      1600000.   5.0380E-15        0.           0.896       24.126        -0.          1
    30.000      1600000.   5.0380E-15        0.           0.851       28.497         0.          1
    35.000      1600000.   5.0380E-15        0.           0.799       32.626        -0.          1
    40.000      1600000.   5.0380E-15        0.           0.741       36.479         0.          1
    45.000      1600000.   5.0380E-15        0.           0.676       40.022        -0.          1
    50.000      1600000.   5.0380E-15        0.           0.605       43.227         0.          1
    55.000      1600000.   5.0380E-15        0.           0.529       46.065        -0.          1
    60.000      1600000.   5.0380E-15        0.           0.449       48.512         0.          1
    65.000      1600000.   5.0380E-15        0.           0.365       50.549        -0.          1
    70.000      1600000.   5.0380E-15        0.           0.278       52.157         0.          1
    75.000      1600000.   5.0380E-15        0.           0.188       53.323        -0.          1
    80.000      1600000.   5.0380E-15        0.           0.097       54.037         0.          1
    85.000      1600000.   5.0380E-15        0.           0.005       54.294        -0.          1
    90.000      1600000.   5.0380E-15        0.          -0.087       54.090         0.          1
    95.000      1600000.   5.0380E-15        0.          -0.178       53.428        -0.          1
   100.000      1600000.   5.0380E-15        0.          -0.268       52.313         0.          1
   105.000      1600000.   5.0380E-15        0.          -0.355       50.755        -0.          1
   110.000      1600000.   5.0380E-15        0.          -0.440       48.766         0.          1
   115.000      1600000.   5.0380E-15        0.          -0.520       46.364        -0.          1
   120.000      1600000.   5.0380E-15        0.          -0.597       43.570         0.          1
   125.000      1600000.   5.0380E-15        0.          -0.668       40.406        -0.          1
   130.000      1600000.   5.0380E-15        0.          -0.734       36.900         0.          1
   135.000      1600000.   5.0380E-15        0.          -0.793       33.081        -0.          1
   140.000      1600000.   5.0380E-15        0.          -0.846       28.981         0.          1
   145.000      1600000.   5.0380E-15        0.          -0.891       24.636        -0.          1
   150.000      1600000.   5.0380E-15        0.          -0.929       20.083         0.          1
   155.000      1600000.   5.0380E-15        0.          -0.959       15.359        -0.          1
   160.000      1600000.   5.0380E-15        0.          -0.981       10.505         0.          1
   165.000      1600000.   5.0380E-15        0.          -0.995        5.562        -0.          1
   170.000      1600000.   5.0380E-15        0.          -1.000        0.571         0.          1
   175.000      1600000.   5.0380E-15        0.          -0.997       -4.424        -0.          1
   180.000      1600000.   5.0380E-15        0.          -0.985       -9.381         0.          1
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185.000    1600000.   5.0380E-15     -0.       -0.965    -14.259     -0.     1
190.000    1600000.   5.0380E-15     -0.       -0.937    -19.017      0.     1
195.000    1600000.   5.0380E-15     -0.       -0.900    -23.613     -0.     1
200.000    1600000.   5.0380E-15     -0.       -0.857    -28.009      0.     1
205.000    1600000.   5.0380E-15     -0.       -0.806    -32.167     -0.     1
210.000    1600000.   5.0380E-15     -0.       -0.748    -36.053      0.     1
215.000    1600000.   5.0380E-15     -0.       -0.683    -39.634     -0.     1
220.000    1600000.   5.0380E-15     -0.       -0.613    -42.878      0.     1
225.000    1600000.   5.0380E-15     -0.       -0.538    -45.760     -0.     1
230.000    1600000.   5.0380E-15     -0.       -0.458    -48.253      0.     1
235.000    1600000.   5.0380E-15     -0.       -0.375    -50.338     -0.     1
240.000    1600000.   5.0380E-15     -0.       -0.288    -51.995      0.     1
245.000    1600000.   5.0380E-15     -0.       -0.199    -53.213     -0.     1
250.000    1600000.   5.0380E-15     -0.       -0.108    -53.979      0.     1
255.000    1600000.   5.0380E-15     -0.       -0.016    -54.288     -0.     1
260.000    1600000.   5.0380E-15     -0.        0.076    -54.137      0.     1
265.000    1600000.   5.0380E-15     -0.        0.168    -53.527     -0.     1
270.000    1600000.   5.0380E-15     -0.        0.258    -52.463      0.     1
275.000    1600000.   5.0380E-15     -0.        0.345    -50.955     -0.     1
280.000    1600000.   5.0380E-15     -0.        0.430    -49.015      0.     1
285.000    1600000.   5.0380E-15     -0.        0.511    -46.659     -0.     1
290.000    1600000.   5.0380E-15     -0.        0.588    -43.908      0.     1
295.000    1600000.   5.0380E-15     -0.        0.660    -40.785     -0.     1
300.000    1600000.   5.0380E-15     -0.        0.726    -37.317      0.     1
305.000    1600000.   5.0380E-15     -0.        0.786    -33.532     -0.     1
310.000    1600000.   5.0380E-15     -0.        0.840    -29.463      0.     1
315.000    1600000.   5.0380E-15     -0.        0.886    -25.144     -0.     1
320.000    1600000.   5.0380E-15     -0.        0.925    -20.612      0.     1
325.000    1600000.   5.0380E-15     -0.        0.956    -15.906     -0.     1
330.000    1600000.   5.0380E-15     -0.        0.979    -11.065      0.     1
335.000    1600000.   5.0380E-15     -0.        0.994     -6.130     -0.     1
340.000    1600000.   5.0380E-15     -0.        1.000     -1.143      0.     1
345.000    1600000.   5.0380E-15     -0.        0.997      3.854     -0.     1
350.000    1600000.   5.0380E-15     -0.        0.987      8.818      0.     1
355.000    1600000.   5.0380E-15     -0.        0.968     13.707     -0.     1
360.000    1600000.   5.0380E-15     -0.        0.940     18.481      0.     1
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     OI              HP               EC             GAM             GAMQ             OM
     0.          1600000.        0.              1.0000E 00      0.              0.

 D PHI MIN       D PHI PRINT                                                        RECORD ID
 5.0000E-02       5.0000E 00                                                        22230100.

                                 DELTA                          END DERIVATIVES                                       NT
                                 1.1942E-01                         -0.006        0.940         -0.            79

                 TH PER BD       WEIGHT THETA H1                MAX THETAS
                 0.              0.                                  1.000       54.294          0.

                                 WEIGHT THETA H2                MAX DERIVATIVES
                                -0.                                  0.018        1.000          0.
     OI              HP             EC             GAM          GAMQ          OM
     0.           1600000.       0.025000       1.0000E 00     0.            0.

 D PHI MIN       D PHI PRINT                                                 RECORD ID
 5.0000E-02       5.0000E 00                                                 22230101.

    PHI              H         RHO               GAMMA      ALPHA H1     ALPHA H2     ALPHA PA      N
     5.000        1602088.    4.9958E-15         0.122        0.874        4.871       -0.122       8
    10.000        1608341.    4.8720E-15         0.243        0.740        9.704       -0.244       1
    15.000        1618717.    4.6739E-15         0.362        0.600       14.460       -0.366       1
    20.000        1633147.    4.4132E-15         0.479        0.455       19.108       -0.488       1
    25.000        1651539.    4.1042E-15         0.592        0.307       23.621       -0.611       1
    30.000        1673771.    3.7625E-15         0.701        0.156       27.978       -0.734       1
    35.000        1699695.    3.4034E-15         0.805        0.005       32.167       -0.859       1
    40.000        1729139.    3.0412E-15         0.903       -0.145       36.183       -0.986       1
    45.000        1761906.    2.6879E-15         0.995       -0.293       40.024       -1.115       1
    50.000        1797773.    2.3527E-15         1.080       -0.438       43.695       -1.247       1
    55.000        1836495.    2.0423E-15         1.157       -0.577       47.207       -1.384       1
    60.000        1877804.    1.7606E-15         1.225       -0.710       50.569       -1.525       1
    65.000        1921412.    1.5095E-15         1.285       -0.836       53.795       -1.672       1
    70.000        1967010.    1.2889E-15         1.335       -0.954       56.900       -1.825       1
    75.000        2014273.    1.0975E-15         1.375       -1.063       59.896       -1.985       1
    80.000        2062857.    9.3334E-16         1.405       -1.162       62.797       -2.152       1
    85.000        2112407.    7.9366E-16         1.424       -1.252       65.616       -2.328       1
    90.000        2162553.    6.7569E-16         1.432       -1.332       68.364       -2.512       1
    95.000        2212918.    5.7662E-16         1.430       -1.401       71.051       -2.705       1
   100.000        2263118.    4.9379E-16         1.417       -1.460       73.686       -2.907       1
   105.000        2312764.    4.2478E-16         1.393       -1.508       76.277       -3.117       1
   110.000        2361468.    3.6743E-16         1.358       -1.546       78.830       -3.337       1
   115.000        2408844.    3.1985E-16         1.312       -1.573       81.351       -3.565       1
   120.000        2454511.    2.8044E-16         1.256       -1.589       83.844       -3.802       1
   125.000        2498098.    2.4785E-16         1.190       -1.597       86.312       -4.047       1
   130.000        2539249.    2.2094E-16         1.115       -1.594       88.760       -4.300       1
   135.000        2577620.    1.9877E-16         1.031       -1.583       91.187       -4.560       1
   140.000        2612891.    1.8058E-16         0.939       -1.564       93.597       -4.826       8
   145.000        2644765.    1.6574E-16         0.839       -1.537       95.989       -5.099       2
   150.000        2672969.    1.5375E-16         0.732       -1.503       98.365       -5.376       1
   155.000        2697260.    1.4420E-16         0.619       -1.463      100.722       -5.658       1
   160.000        2717431.    1.3677E-16         0.502       -1.418      103.063       -5.943       1
   165.000        2733305.    1.3123E-16         0.380       -1.369      105.384       -6.231       1
   170.000        2744744.    1.2739E-16         0.255       -1.317      107.685       -6.520       1
   175.000        2751647.    1.2514E-16         0.128       -1.262      109.964       -6.810       1
   180.000        2753955.    1.2439E-16         0.000       -1.206      112.220       -7.099       1
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185.000   2751647.   1.2514E-16   -0.128   -1.150   114.450    -7.386   1
190.000   2744744.   1.2739E-16   -0.255   -1.095   116.651    -7.671   1
195.000   2733305.   1.3123E-16   -0.380   -1.041   118.821    -7.952   1
200.000   2717431.   1.3677E-16   -0.502   -0.990   120.957    -8.228   1
205.000   2697261.   1.4420E-16   -0.619   -0.942   123.055    -8.498   1
210.000   2672969.   1.5375E-16   -0.732   -0.899   125.112    -8.760   1
215.000   2644765.   1.6574E-16   -0.839   -0.862   127.124    -9.015   1
220.000   2612892.   1.8058E-16   -0.939   -0.830   129.085    -9.260   1
225.000   2577620.   1.9877E-16   -1.031   -0.805   130.992    -9.495   1
230.000   2539249.   2.2094E-16   -1.115   -0.788   132.837    -9.718   1
235.000   2498099.   2.4785E-16   -1.190   -0.779   134.614    -9.928   1
240.000   2454511.   2.8044E-16   -1.256   -0.777   136.316   -10.125   1
245.000   2408844.   3.1985E-16   -1.312   -0.785   137.933   -10.306   1
250.000   2361469.   3.6742E-16   -1.358   -0.801   139.454   -10.471   1
255.000   2312765.   4.2478E-16   -1.393   -0.826   140.867   -10.619   1
260.000   2263118.   4.9379E-16   -1.417   -0.860   142.158   -10.747   1
265.000   2212918.   5.7662E-16   -1.430   -0.903   143.308   -10.855   1
270.000   2162553.   6.7569E-16   -1.432   -0.954   144.297   -10.940   1
275.000   2112408.   7.9366E-16   -1.424   -1.013   145.100   -11.001   1
280.000   2062858.   9.3333E-16   -1.405   -1.079   145.688   -11.036   1
285.000   2014274.   1.0975E-15   -1.375   -1.151   146.025   -11.041   1
290.000   1967011.   1.2889E-15   -1.335   -1.229   146.074   -11.015   1
295.000   1921412.   1.5095E-15   -1.285   -1.310   145.787   -10.953   1
300.000   1877804.   1.7606E-15   -1.225   -1.394   145.114   -10.853   1
305.000   1836495.   2.0423E-15   -1.157   -1.479   143.997   -10.711   1
310.000   1797773.   2.3527E-15   -1.080   -1.563   142.373   -10.523   1
315.000   1761906.   2.6879E-15   -0.995   -1.644   140.176   -10.284   1
320.000   1729140.   3.0412E-15   -0.903   -1.720   137.340    -9.991   1
325.000   1699695.   3.4034E-15   -0.805   -1.789   133.799    -9.639   1
330.000   1673771.   3.7624E-15   -0.701   -1.849   129.492    -9.227   1
335.000   1651539.   4.1042E-15   -0.592   -1.898   124.369    -8.750   1
340.000   1633147.   4.4132E-15   -0.479   -1.934   118.392    -8.210   1
345.000   1618717.   4.6739E-15   -0.362   -1.955   111.543    -7.605   1
350.000   1608341.   4.8720E-15   -0.243   -1.961   103.824    -6.937   1
355.000   1602088.   4.9958E-15   -0.122   -1.950    95.259    -6.212   1
360.000   1600000.   5.0380E-15   -0.000   -1.923    85.897    -5.434   1
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      OI             HP              EC             GAM          GAMQ          OM
      0.          1600000.       0.025000       1.0000E 00     0.            0.
 D PHI MIN      D PHI PRINT                                                  RECORD ID
 5.0000E-02      5.0000E 00                                                  22230101.
                               DELTA                      END DERIVATIVES                          NT
                               5.8469E 0C                       0.031        -1.923        0.185   87
                TH PER BD      WEIGHT THETA H1            MAX THETAS
                2.1596E 01    -8.7613E-07                       2.643       144.739       12.440
                               WEIGHT THETA H2            MAX DERIVATIVES
                               6.3257E-02                       0.031         1.923        0.185
      TAPE RECORD      WEIGHT PAR     WEIGHT GAMMA    WEIGHT HI       WEIGHT H2
       22230101.       1.000000E 00   0.              0.              0.

 SCALING - MIN VALUE    MIN COUNTS      MAX VALUE       MAX COUNTS      EDGE COUNTS     CARD FIELD
 FUNCTION     -30.000          200.        30.000             800.             850.            2.
 ARGUMENT          0.          100.       360.000             820.             850.            1.

 DECK ID -      0.

 D WHS   5.846886 -0.000001  0.063257

 FNCTN  0.         -0.000145 -0.001167 -0.003979 -0.009561 -0.018978 -0.033398 -0.054080 -0.082365 -0.119653
    11 -0.167370 -0.226938 -0.299741 -0.367090 -0.490200 -0.610158 -0.747910 -0.904235 -1.079742 -1.274852
    21 -1.489798 -1.724617 -1.979151 -2.253047 -2.545762 -2.856563 -3.184539 -3.528604 -3.887509 -4.259852
    31 -4.644092 -5.038556 -5.441461 -5.850924 -6.264979 -6.681594 -7.098686 -7.514138 -7.925817 -8.331587
    41 -8.729327 -9.116940 -9.492370 -9.853608 -10.19871 -10.52577 -10.83298 -11.11858 -11.38085 -11.61814
    51 -11.82881 -12.01126 -12.16383 -12.28485 -12.37254 -12.42501 -12.44018 -12.41578 -12.34928 -12.23790
    61 -12.07857 -11.86800 -11.60273 -11.27925 -10.89417 -10.44445 -9.927682 -9.342381 -8.688284 -7.966612
    71 -7.180254 -6.333826 -5.433610

   ARG  0.         5.000000  10.00000  15.00000  20.00000  25.00000  30.00000  35.00000  40.00000  45.00000
    11  50.00000  55.00000  60.00000  65.00000  70.00000  75.00000  80.00000  85.00000  90.00000  95.00000
    21  100.0000  105.0000  110.0000  115.0000  120.0000  125.0000  130.0000  135.0000  140.0000  145.0000
    31  150.0000  155.0000  160.0000  165.0000  170.0000  175.0000  180.0000  185.0000  190.0000  195.0000
    41  200.0000  205.0000  210.0000  215.0000  220.0000  225.0000  230.0000  235.0000  240.0000  245.0000
    51  250.0000  255.0000  260.0000  265.0000  270.0000  275.0000  280.0000  285.0000  290.0000  295.0000
    61  300.0000  305.0000  310.0000  315.0000  320.0000  324.9999  329.9999  334.9999  339.9999  344.9999
    71  349.9999  354.9999  359.9999
```

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      TAPE RECORD     WEIGHT PAR     WEIGHT GAMMA    WEIGHT HI      WEIGHT H2
       22230101.      0.             1.000000E 00    0.             0.

SCALING - MIN VALUE   MIN COUNTS     MAX VALUE       MAX COUNTS     EDGE COUNTS    CARD FIELD
FUNCTION    -30.000         200.        30.000             800.           850.           3.
ARGUMENT      0.            100.       360.000             820.           850.           0.

DECK ID -     0.

D WHS  5.846886 -0.000001  0.063257

FNCTN  0.        0.121808  0.242756  0.361990  0.478663  0.591944  0.701020  0.805100  0.903424  0.995262
   11  1.079924  1.156762  1.225175  1.284618  1.334599  1.374692  1.404536  1.423841  1.432394  1.430060
   21  1.416784  1.392598  1.357619  1.312053  1.256192  1.190419  1.115199  1.031083  0.938703  0.838765
   31  0.732047  0.619390  0.501694  0.379905  0.255012  0.128031  0.000001 -0.128029 -0.255010 -0.379904
   41 -0.501693 -0.619389 -0.732046 -0.828764 -0.938702 -1.031082 -1.115198 -1.190418 -1.256192 -1.312052
   51 -1.357619 -1.392597 -1.416784 -1.430060 -1.432394 -1.423842 -1.404536 -1.374692 -1.334599 -1.284618
   61 -1.225176 -1.156762 -1.079925 -0.995263 -0.903425 -0.805101 -0.701021 -0.591945 -0.478664 -0.361991
   71 -0.242757 -0.121809 -0.000001
```

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